# Ćwiczenia 3-4

1. Różne style matematyczne w trybie matematycznym:

Tryb kaligraficzny (tylko duże litery)
$$\mathcal{ABC}$$

Pogrubione litery bez wypełnienia (tylko duże litery)

$$\mathbb{ABC}$$

Styl gotycki
$$\mathfrak{ABCabc}$$

Styl bezszeryfowy
$$\mathsf{ABCabc}$$

Pogrubione litery z wypełnieniem
$$\mathbf{ABCabc}$$

```
Tryb kaligraficzny (tylko duże litery)
$$\mathcal{A B C}$$
Pogrubione litery bez wypełnienia (tylko duże litery)
$$\mathbb{A B C}$$
Styl gotycki
$$\mathfrak{A B C abc}$$
Styl bezszeryfowy
$$\mathsf{A B C abc}$$
Pogrubione litery z wypełnieniem
$$\mathbf{A B C abc}$$
```

2. Odstępy w trybie matematycznym:

$$f(x) = x^2\!+3x\!+2$$
$$f(x) = x^2+3x+2$$
$$f(x) = x^2\,+3x\,+2$$
$$f(x) = x^2\:+3x\:+2$$
$$f(x) = x^2\;+3x\;+2$$
$$f(x) = x^2\ +3x\ +2$$
$$f(x) = x^2\quad+3x\quad+2$$
$$f(x) = x^2\qquad+3x\qquad+2$$

```
$$f(x) = x^2\! +3x\! +2 $$
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$$f(x) = x^2\, +3x\, +2 $$
$$f(x) = x^2\: +3x\: +2 $$
$$f(x) = x^2\; +3x\; +2 $$
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$$f(x) = x^2\quad +3x\quad +2 $$
$$f(x) = x^2\qquad +3x\qquad +2$$
```

3. Automatyczne nawiasy \left \right

$$2+\left( \frac{3}{4-x}\right)^2$$
$$\left[ 2-3\cdot \left(2-x\right)^3\right]^{y-2}$$
$$\left\{\frac{2}{3}-\cos^2 x+\left[ 4-\left(\frac{2-3x}{3-9x} \right)^3\right]^2 \right\}^7$$

$$2 + \left( \frac{3}{4-x} \right)^2$$

$$\left[ 2 - 3 \cdot (2-x)^3 \right]^{y-2}$$

$$\left\{ \frac{2}{3} - \cos^2 x + \left[ 4 - \left( \frac{2-3x}{3-9x} \right)^3 \right]^2 \right\}^7$$

4. Akcenty

$$\hat{a} \ \check{b} \ \acute{c} \ \dot{e} \ \vec{f}$$

$$\hat{a} \ \check{b} \ \acute{c} \ \dot{e} \ \vec{f}$$

5. Długie zaznaczenia:

$$\overline{abc} \ \underline{efg} \ \widetilde{AFR}$$

$$\overline{abc} \ \underline{efg} \ \widetilde{AFR}$$

6. Łamanie równań https://www.overleaf.com/learn/latex/Aligning%20equations%20with%20amsmath

$$\begin{split} y &= (x-2)^2 \\ &= x^2 - 4x + 4 \end{split}$$

```
\begin{equation*}
\begin{split}
y & = (x-2)^2 \\
 & = x^2-4x+4
\end{split}
\end{equation*}
```

7. Rozpisywanie na przypadki https://pt.overleaf.com/learn/latex/Questions/How__to__handle__mathem atical__expressions__involving__case__statements

$$|x-2| = \begin{cases} x-2, & \text{jeśli } x \geqslant 2 \\ -(x-2), & \text{jeśli } x < 2 \end{cases}$$

```
$$
|x-2|=\begin{cases}
         x-2, & \text{jeśli $x\geqslant 2$}\\
         -(x-2), & \text{jeśli $x<2$}
       \end{cases}
$$
```

8. Strzałki:

$$f : \mathbb{R} \mapsto \mathbb{R}$$
$$g : \mathbb{Z} \rightarrow \mathbb{Z}$$
$$x \Leftarrow r$$

```
$$f: \mathbb{R}\mapsto \mathbb{R}$$
$$g: \mathbb{Z}\rightarrow \mathbb{Z}$$
$$x \Leftarrow r$$
```

9. Środowisko `array`

```
$$\begin{array}{|l|cr}
left1 & center1 & right1\\
\hline
d & e & f
\end{array}$$
```

$$\begin{array}{|l|cr}
left1 & center1 & right1\\
\hline
d & e & f
\end{array}$$

```
$$\begin{array}{lcl}
z & = & a \\
& = & a \\
f(x,y,z) & = & x + y + z
\end{array}$$
```

$$\begin{array}{lcl}
z & = & a \\
& = & a \\
f(x,y,z) & = & x + y + z
\end{array}$$

```
$$\chi(x) =
  \left|
    \begin{array}{ccc}
      x-a   &-b  &-c  \\
      -d    &x-e &-f  \\
      -g    &-h  &x-i
    \end{array}
  \right|$$
```

$$\chi(x) = \left| \begin{array}{ccc}
x - a & -b & -c \\
-d & x - e & -f \\
-g & -h & x - i
\end{array} \right|$$

```
$$\left[\begin{array}{c|c|c|c}
A &Ab &\cdots &A^{n-1}b
\end{array}\right]$$
```

$$\left[ \begin{array}{c|c|c|c} A & Ab & \cdots & A^{n-1}b \end{array} \right]$$

```
$$\left[\begin{array}{cccc|c}
a_{11}&a_{12}&\cdots &a_{1n}&b_1\\
a_{21}&a_{22}&\cdots &a_{2n}&b_2\\
\vdots & &\ddots &\ \vdots\\
a_{n1}&a_{n2}&\cdots &a_{nn}&b_n\\
\end{array}\right]
$$
```

$$
\left[
\begin{array}{cccc|c}
a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\
a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\
\vdots & & \ddots & \vdots & \\
a_{n1} & a_{n2} & \cdots & a_{nn} & b_n
\end{array}
\right]
$$

```
$$
C=\left[
\begin{array}{c|c}
A &B \\ \hline
C &D
\end{array}\right]$$
```

$$
C = \left[ \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]
$$

10. Poćwicz różne środowiska dedykowane dla macierzy https://www.overleaf.com/learn/latex/Matrices

11. Całki i granice

$$\lim_{n\to\infty} \left(1-\frac{1}{a^n}\right)=1$$

$$\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n=e$$

$$\int_a^b \sin(x)\,dx$$

$$\iint\displaylimits_D (x^2+y-4)\,dx\,dy$$

```
$$\lim_{n\to\infty} \left(1-\frac{1}{a^n}\right)=1$$
$$\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n=e$$
$$\int_a^b \sin(x) \, dx$$
$$\iint\displaylimits_D (x^2+y-4) \, dx\, dy$$
```

Wskazówka: przed indeksami do całki wstaw `\displaylimits`.

13. W jednym pliku postaraj się odwzorować wzory:

Wyrażenie #1

$$\lim_{n\to\infty} \frac{n+1}{n} = 1$$

$$\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$$

$$\lim_{n\to\infty} \frac{2n+5}{n} = 2$$

$$\lim_{n\to\infty} (2n-1) = \infty$$

$$\lim_{n\to\infty} (-n^2+1) = -\infty$$

Wyrażenie #2

$$
\begin{cases}
|z| = |z-4i| \\
\frac{\pi}{4} \le \operatorname{Arg} z < \frac{\pi}{2}
\end{cases}
$$

$$\begin{cases} |z+4| = |z+2-2i| \\ |z| \leq 2 \end{cases}$$

$$\begin{cases} |z-1-i| < \sqrt{2} \\ \operatorname{Arg}(z-1-i) < \frac{\pi}{2} \end{cases}$$

Wyrażenie #3

$$\begin{cases} x + 5y = 2 \\ -3x + 6y = 15 \end{cases}$$

$$\begin{cases} x - y - z = 1 \\ 3x + 4y - 2z = -1 \\ 3x - 2y - 2z = 1 \end{cases}$$

$$\begin{cases} y - 3z + 4v = 0 \\ x - 2z = 0 \\ 3x + 2y - 5v = 2 \\ 4x - 5z = 0 \end{cases}$$

Wyrażenie #4

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 5 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 11 & -2 \\ 6 & -14 \\ -21 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 1 & 3 & 0 \end{bmatrix}$$

Wyrażenie #5

$$\begin{cases} x + 2y + 3z + t = 1 \\ 2x + 4y - z + 2t = 2 \\ 3x + 6y + 10z + 3t = 3 \\ x + y + z + t = 0 \end{cases}$$

$$\begin{cases} x - y + z - 2s + t = 0 \\ 3x + 4y - z + s + 3t = 1 \\ x - 8y + 5z - 9s + t = -1 \end{cases}$$

Wyrażenie #6

$$\begin{vmatrix} -3 & 2 \\ 8 & -5 \end{vmatrix}$$

$$\begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{vmatrix}$$

**Wyrażenie #7**

$$B = \left[\begin{array}{c|cc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ \hline 0 & 2 & 2 & 1 & 2 & 3 \\ 0 & 2 & 2 & 4 & 5 & 6 \\ \hline 0 & 0 & 0 & 3 & 3 & 1 \\ 0 & 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 1 & 3 & 3 \end{array}\right]$$

**Wyrażenie #8**

$$\int_1^\infty \frac{dx}{(x+2)^2}$$

$$\int_{-\infty}^0 \frac{dx}{x^2+4}$$

$$\int_{-\infty}^\infty x^2 e^{-x^3}\, dx$$

$$\int_1^\infty \frac{dx}{\sqrt[3]{3x+5}}$$

$$\int_{-1}^0 \frac{dx}{\sqrt[5]{x^2}}$$

$$\int_2^3 \frac{dx}{x^2-3x}$$

**Wyrażenie #9**

$$\log_{\sqrt{5}} 5\sqrt[3]{5}$$

$$\log_{\sqrt[3]{3}} 27$$

$$\log_2 8\sqrt{2}$$

$$\log_{\frac{1}{3}} 81\sqrt{3}$$

$$3^{2+\log_3 4}$$

$$2^{5-\frac{1}{3}\log_2 27}$$

$$\sqrt{10^{2+\frac{1}{2}\log 16}}$$

**Wyrażenie #10**

$$\int \frac{x^2\, dx}{\sqrt{4-x^2}}$$

$$\int \frac{x^3\, dx}{\sqrt{25+x^2}}$$

$$\int \sqrt{x^2-36}\, dx$$

$$\int \sqrt{3+x^2}\, dx$$

**Wyrażenie #11**

$$\lim_{n\to\infty} \left(\sqrt{n+6\sqrt{n}+1}-\sqrt{n}\right)$$

$$\lim_{n\to\infty} \frac{1+\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^n}}$$

**Wyrażenie #12**

$$d_n = \cos\frac{\pi}{2n}$$

$$e_n = \sqrt[n]{5^n + 6^n}$$

$$f_n = \frac{n!(2n)!}{(3n)!}$$

**Wyrażenie #13**

$$\lim_{n\to\infty} \left(1 + \frac{6}{n}\right)^n$$

$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{n+1}$$

$$\lim_{n\to\infty} \left(\frac{n+3}{n}\right)^{n+3}$$

$$\lim_{n\to\infty} \left(1 - \frac{2}{n}\right)^{-n}$$

**Wyrażenie #14**

$$\sum_{n=1}^{\infty} (-1)^{n+1}(2n-1)$$

$$\sum_{n=1}^{\infty} \sin \frac{2\pi}{3^n} \cos \frac{4\pi}{3^n}$$

$$\sum_{n=2}^{\infty} \left(\left(\sqrt[n]{n} - \sqrt[n+1]{n+1}\right)\right)$$