①

$$A = \begin{vmatrix} 2 & 3 & 5 \\ -1 & 4 & 6 \\ 3 & -2 & 7 \end{vmatrix}$$

Rozwinięciem Laplace'a

po 1-mu wierszu:

$$\det A = 2\cdot(-1)^{1+1}\begin{vmatrix} 4 & 6 \\ -2 & 7 \end{vmatrix} + 3\cdot(-1)^{1+2}\begin{vmatrix} -1 & 6 \\ 3 & 7 \end{vmatrix} +$$

$$+ 5\cdot(-1)^{1+3}\begin{vmatrix} -1 & 4 \\ 3 & -2 \end{vmatrix} = 2(28+12) - 3(-7-18) +$$

$$+ 5(2-12) = 80 + 75 - 50 = 105$$

po 1-ej kolumnie:

$$\det A = 2\cdot(-1)^{1+1}\begin{vmatrix} 4 & 6 \\ -2 & 7 \end{vmatrix} - 1\cdot(-1)^{2+1}\begin{vmatrix} 3 & 5 \\ -2 & 7 \end{vmatrix} +$$

$$+ 3\cdot(-1)^{3+1}\begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix} = 2\cdot(28+12) + 1\cdot(21+10) + 3(18-20)$$

$$= 80 + 31 - 6 = 105.$$

$$A = \begin{pmatrix} 0 & 1 & 2 & 7 \\ 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

Rozwinięcie Laplace'a po pierwszemu wierszu

$\det A =$

$$0 + 1\cdot(-1)^{1+2}\begin{vmatrix} 1 & 3 & 4 \\ 5 & 7 & 8 \\ -1 & -1 & 1 \end{vmatrix} + 2\cdot(-1)^{1+3}\begin{vmatrix} 1 & 2 & 4 \\ 5 & 6 & 8 \\ -1 & 1 & 1 \end{vmatrix}$$

$$+ 7\cdot(-1)^{1+4}\begin{vmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ -1 & 1 & -1 \end{vmatrix}$$

$$\left|\begin{array}{ccc|cc} 1 & 3 & 4 & 1 & 3 \\ 5 & 7 & 8 & 5 & 7 \\ -1 & -1 & +1 & -1 & -1 \end{array}\right. = +7 - 24 - 20 + 28 + 8 - 15 = -16$$

$$\left|\begin{array}{ccc|cc} 1 & 2 & 4 & 1 & 2 \\ 5 & 6 & 8 & 5 & 6 \\ -1 & 1 & 1 & -1 & 1 \end{array}\right. = 6 - 16 + 20 + 24 - 8 - 10 = 16$$

$$\left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 5 & 6 & 7 & 5 & 6 \\ -1 & 1 & -1 & -1 & 1 \end{array}\right. = -6 - 14 + 15 + 18 - 7 + 10 = 16$$

$$\det A = 0 + (-1)\cdot(-16) + 2\cdot 16 - 7\cdot 16 = -4\cdot 16 = -64.$$

Macierzy odwrotne:

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

$$\det A = \left| \begin{matrix} 1 & 3 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{matrix} \right| \begin{matrix} 1 & 3 \\ 1 & 1 \\ 1 & 0 \end{matrix} = 2+3+0-1-0-6 = -2$$

$$(-1)^{i+j} A_{ij}$$

$$A^{-1} = -\frac{1}{2} \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} \end{pmatrix}^T =$$

$$= -\frac{1}{2} \begin{pmatrix} 2-0 & -(2-1) & (0-1) \\ -(6) & 2-1 & -(0-3) \\ 3-1 & -0 & 1-3 \end{pmatrix}^T = -\frac{1}{2} \begin{pmatrix} 2 & -1 & -1 \\ -6 & 1 & 3 \\ 2 & 0 & -2 \end{pmatrix}^T$$

$$= \begin{pmatrix} -1 & 0.5 & 0.5 \\ 3 & -0.5 & -1.5 \\ -1 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} -1 & 3 & -1 \\ 0.5 & -0.5 & 0 \\ 0.5 & -1.5 & 1 \end{pmatrix}$$

Sprawdzamy

$$\begin{pmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} -1 & 3 & -1 \\ 0.5 & -0.5 & 0 \\ 0.5 & -1.5 & 1 \end{pmatrix} = \begin{pmatrix} -1+1.5+0.5 & 3-1.5-1.5 & -1+1 \\ -1+0.5+0.5 & 3-0.5-1.5 & -1+1 \\ -1+1 & 3-3 & -1+2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \checkmark$$

$$B = \begin{pmatrix} 3 & 2 & 3 \\ -4 & -4 & -5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\det B = 1 \cdot \begin{vmatrix} 2 & 3 \\ -4 & -5 \end{vmatrix} - 1 \cdot \begin{vmatrix} 3 & 3 \\ -4 & -5 \end{vmatrix} + 1 \cdot \begin{vmatrix} 3 & 2 \\ -4 & -4 \end{vmatrix}$$

$$= (-10+12) - (-15+12) + (-12+8) = 2+3-4 = 1$$

$$B^{-1} = 1 \cdot \begin{pmatrix} \begin{vmatrix} -4 & -5 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} -4 & -5 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} -4 & -4 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ -4 & -5 \end{vmatrix} & -\begin{vmatrix} 3 & 3 \\ -4 & -5 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ -4 & -4 \end{vmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & -4 \end{pmatrix}^T = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 3 \\ 0 & -1 & -4 \end{pmatrix}$$

Sprawdzamy:

$$\begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 3 \\ 0 & -1 & -4 \end{pmatrix} \begin{pmatrix} 3 & 2 & 3 \\ -4 & -4 & -5 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3-4+2 & 2-4+2 & 3-5+2 \\ -3+3 & -2+3 & -3+3 \\ 4-4 & 4-1 & 5-4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \checkmark$$