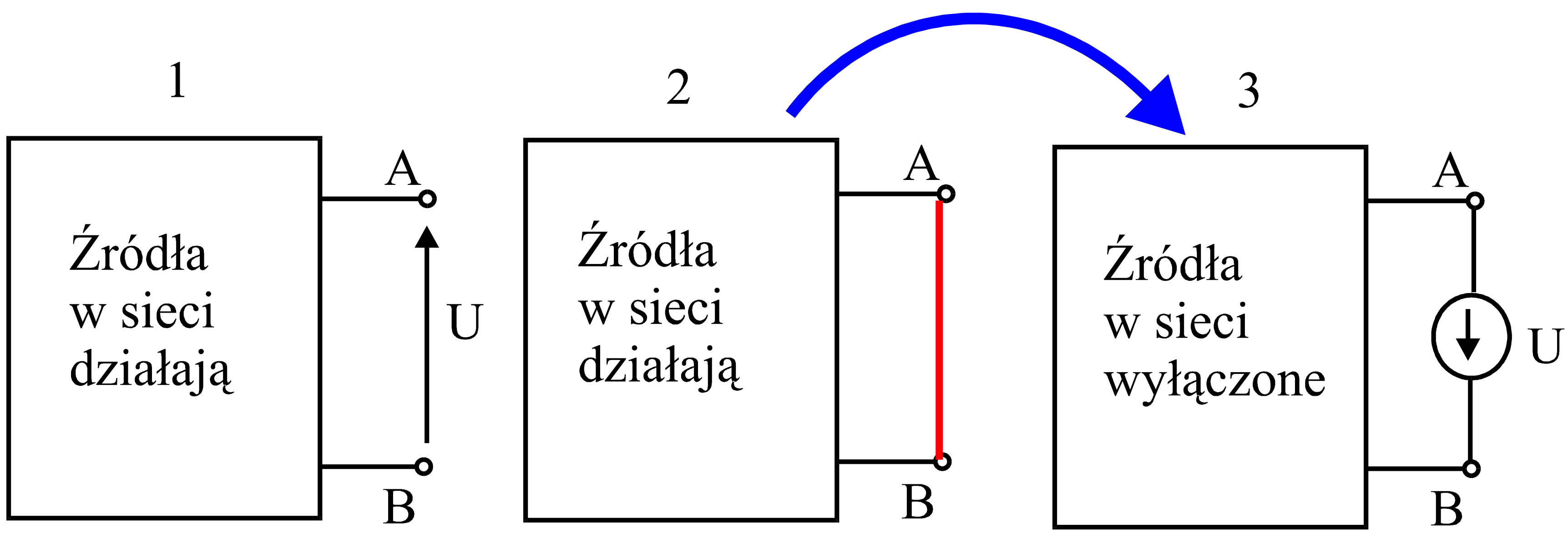
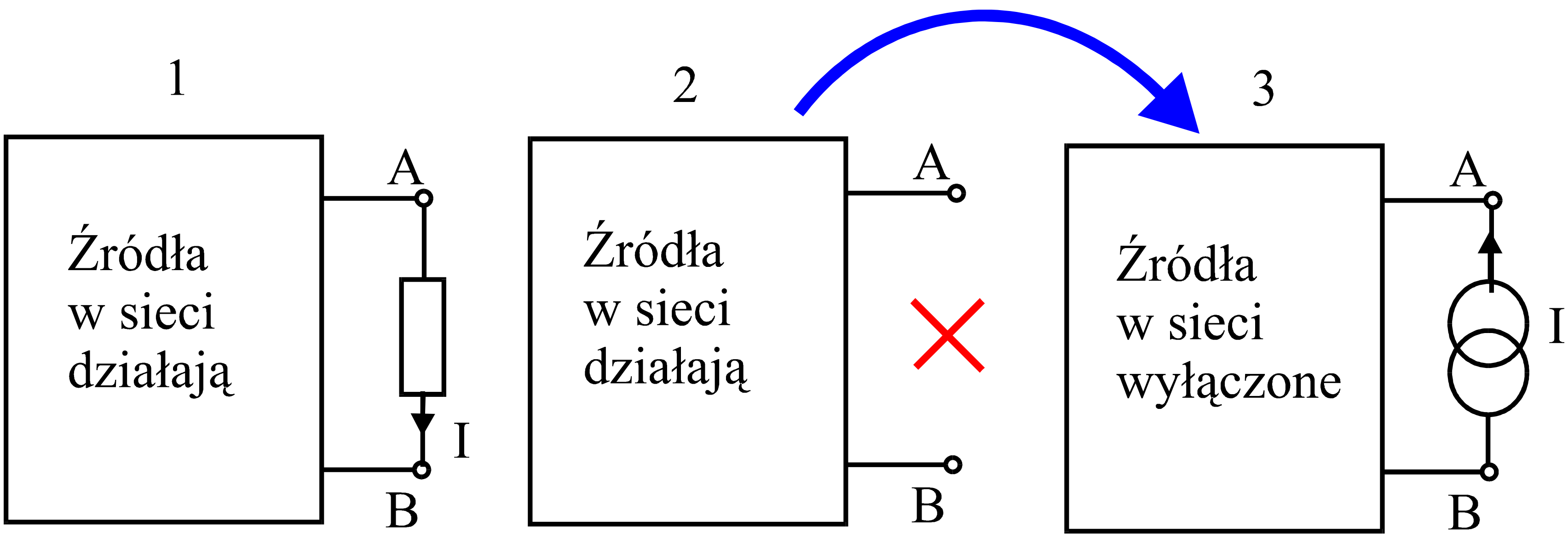
Twierdzenia o przyrostach

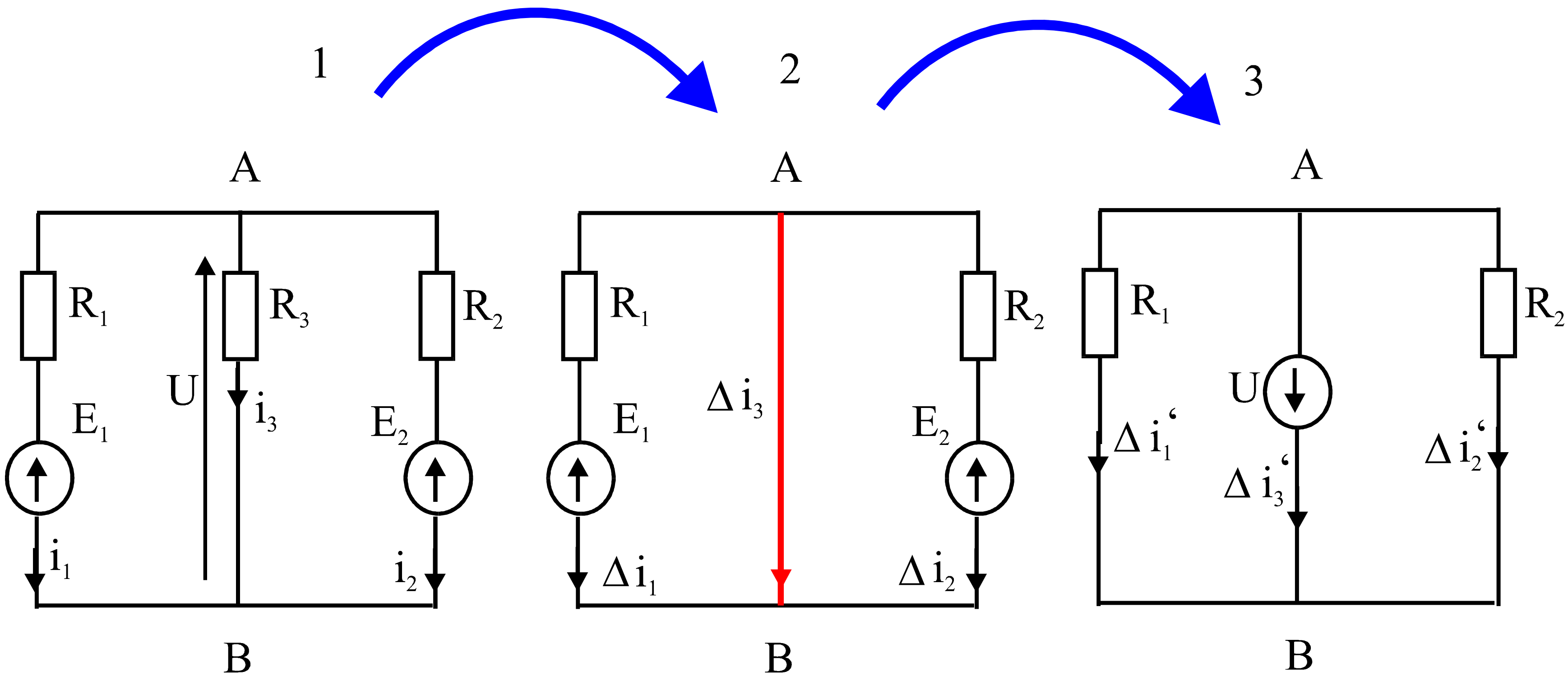
1) Jeżeli w sieci liniowej zewrzemy dwa węzły, między którymi panuje napięcie U, to przyrosty (dodatnie lub ujemne) prądów w gałęziach tej sieci możemy obliczyć włączając między te węzły idealne źródło napięciowe o sile elektromotorycznej równej co do wartości temu napięciu lecz przeciwnym kierunku, przy wyłączeniu wszystkich źródeł pozostałych.



2) Jeżeli w sieci liniowej rozewrzemy gałęź, w której płynie prąd I, to przyrosty (dodatnie lub ujemne) prądów w gałęziach tej sieci możemy obliczyć włączając w tym miejscu idealne źródło prądowe o prądzie zwarcia równym co do wartości temu prądowi lecz przeciwnym kierunku, przy wyłączeniu wszystkich źródeł pozostałych.



Zwarcie



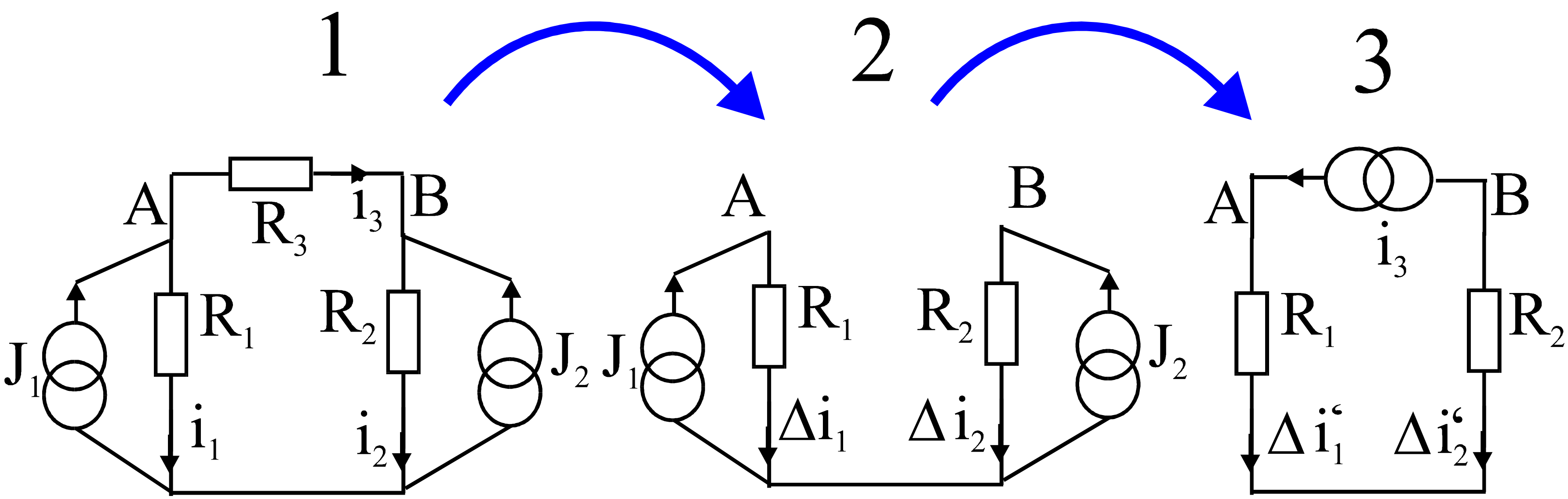
$$i_1 = \frac{U - E_1}{R_1}$$

$$\Delta i_1 = \frac{-E_1}{R_1} - i_1 = -\frac{U}{R_1}$$

$$\Delta i_1' = \frac{-U}{R_1}$$

$$\boxed{\Delta i_1' = \Delta i_1}$$

Rozwarcie

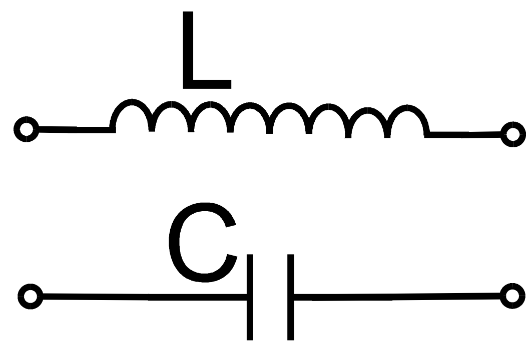


$$i_1 = J_1 - i_3 \qquad \Delta i_1 = J_1 - i_1 = i_3 \qquad \Delta i_1' = i_3$$

$$\boxed{\Delta i_1' = \Delta i_1}$$

Elementy reaktancyjne

- Cewki (induktory)
- Kondensatory



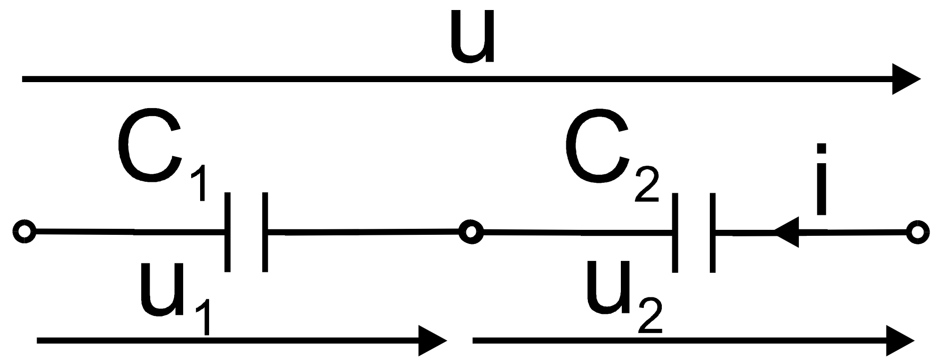
$$\psi = Li$$ Charakterystyka induktora $$u = L\frac{di}{dt}$$

$$e = -\frac{d\psi}{dt}$$ Prawo Faraday'a $$i = \frac{1}{L}\int u dt$$

$$q = Cu$$ Charakterystyka kondensatora $$i = C\frac{du}{dt}$$ $$u = \frac{1}{C}\int i dt$$

Łączenie kondensatorów

- Szeregowe



$$u_1 = \frac{1}{C_1}\int i dt$$

$$u_2 = \frac{1}{C_2}\int i dt$$

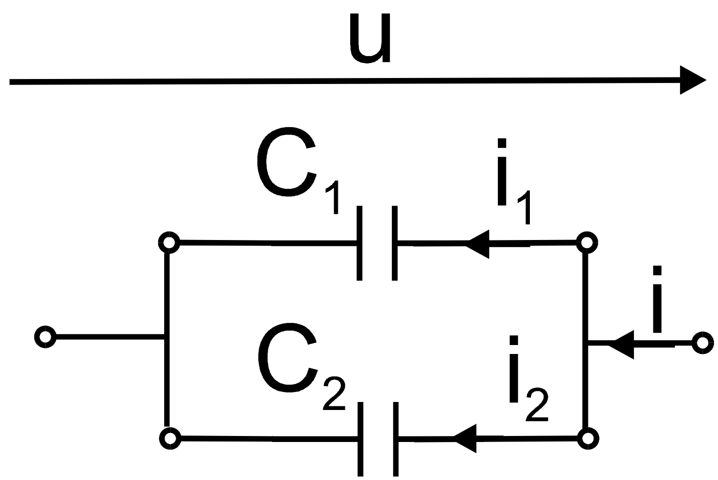
$$u = u_1 + u_2$$

$$u = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int i dt = C_z \int i dt$$

Łączenie kondensatorów

- Równoległe

$$i_1 = C_1 \frac{du}{dt}$$



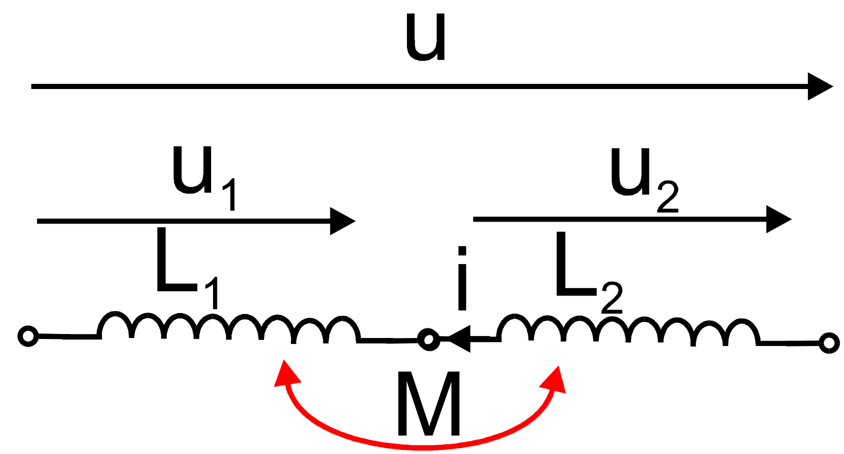
$$i_2 = C_2 \frac{du}{dt}$$

$$i = i_1 + i_2$$

$$i = \left(C_1 + C_2\right)\frac{du}{dt} = C_z \frac{du}{dt}$$

Łączenie cewek

- Szeregowe



$$u_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$$

$$u_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$$

$$u = u_1 + u_2$$

$$u = \left(L_1 + L_2 + 2M\right)\frac{di}{dt} = L_z \frac{di}{dt}$$

$$u\Big|_{M=0} = \left(L_1 + L_2\right)\frac{di}{dt} = L_z \frac{di}{dt}$$

Łączenie cewek

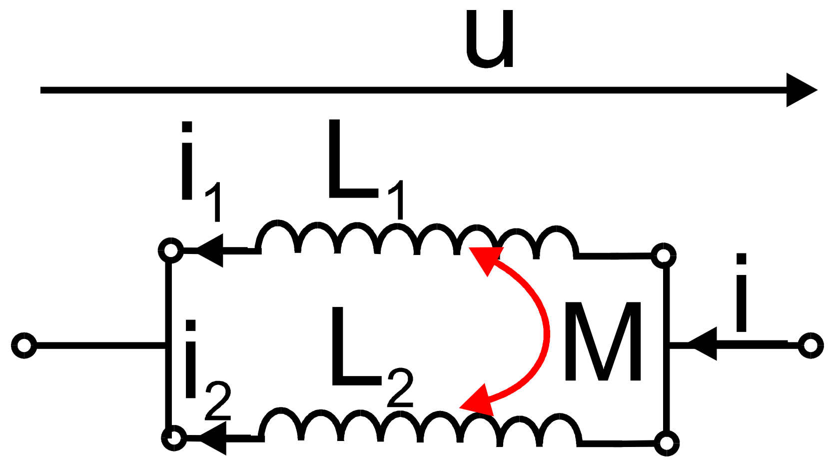
- Równoległe

$$u = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$i = i_1 + i_2$$

$$\Gamma_L = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}$$



$$\frac{di_1}{dt} = \frac{u}{\det \Gamma_L}\left(L_2 - M\right)$$

$$\frac{di_2}{dt} = \frac{u}{\det \Gamma_L}\left(L_1 - M\right)$$

$$u = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \frac{di}{dt} = L_z \frac{di}{dt}$$

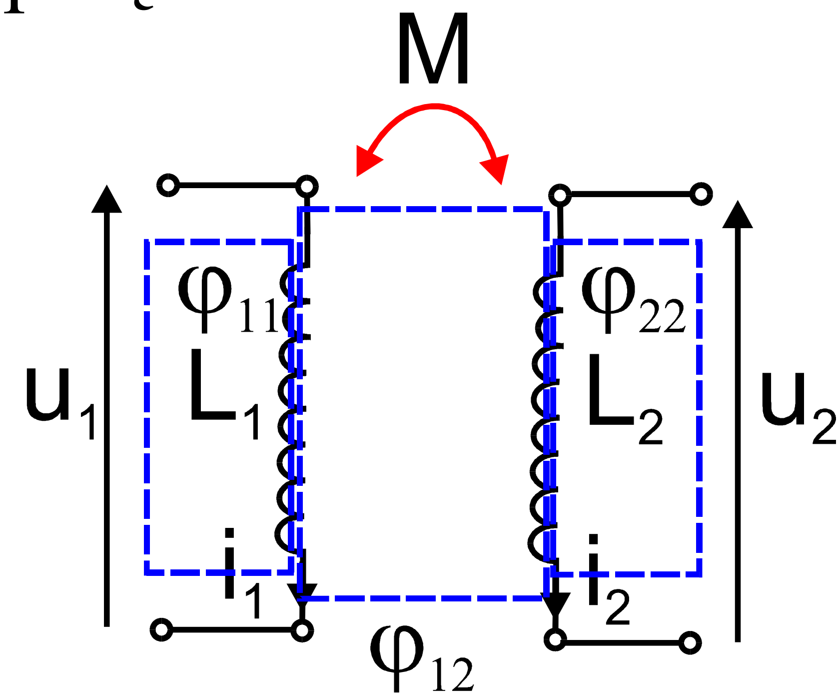
Cewki sprzężone

$$L_{jk} = \frac{n_j \varphi_{jk}}{i_k}$$

$$L_{jk} \frac{di_k}{dt} = n_j \frac{d\varphi_{jk}}{dt}$$

$$u_1 = n_1 \frac{d\varphi_{11}}{dt} + n_1 \frac{d\varphi_{12}}{dt}$$

$$u_2 = n_2 \frac{d\varphi_{22}}{dt} + n_2 \frac{d\varphi_{21}}{dt}$$



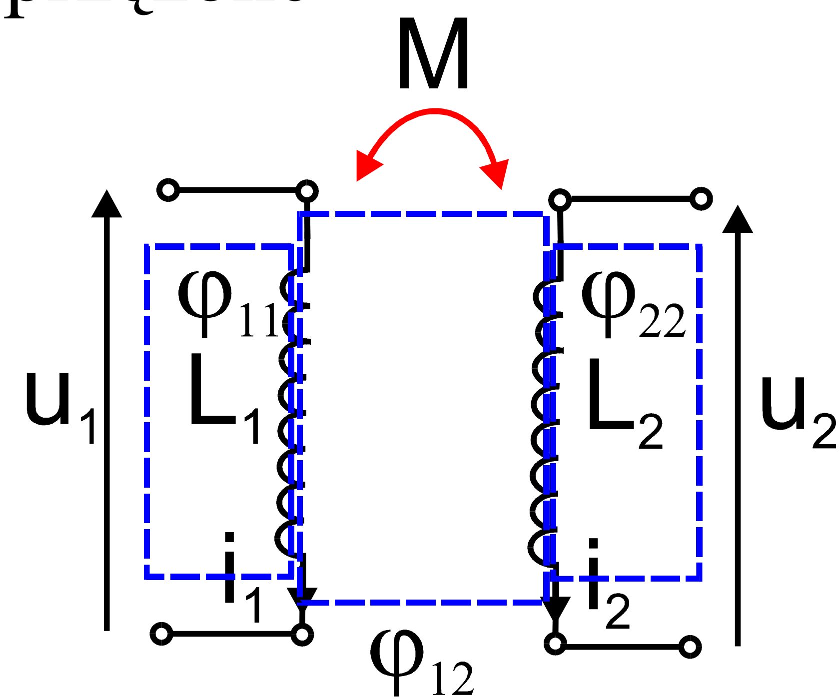
$$\frac{n_1 \varphi_{12}}{i_2} = \frac{n_2 \varphi_{21}}{i_1}$$

$$L_{12} = L_{21}$$

Cewki sprzężone

$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



- Współczynnik sprzężenia

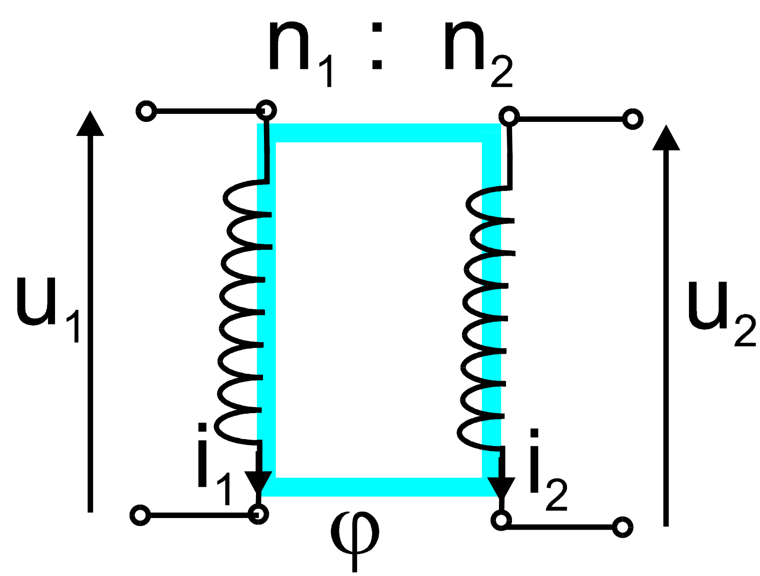
$$k = \frac{|M|}{\sqrt{L_1 L_2}}$$

Transformator idealny

$$u_1 = n_1 \frac{d\varphi}{dt}$$

$$u_2 = n_2 \frac{d\varphi}{dt}$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{n_1}{n_2} = p$$



$$n_1 i_1 + n_2 i_2 = 0$$

$$\Downarrow$$

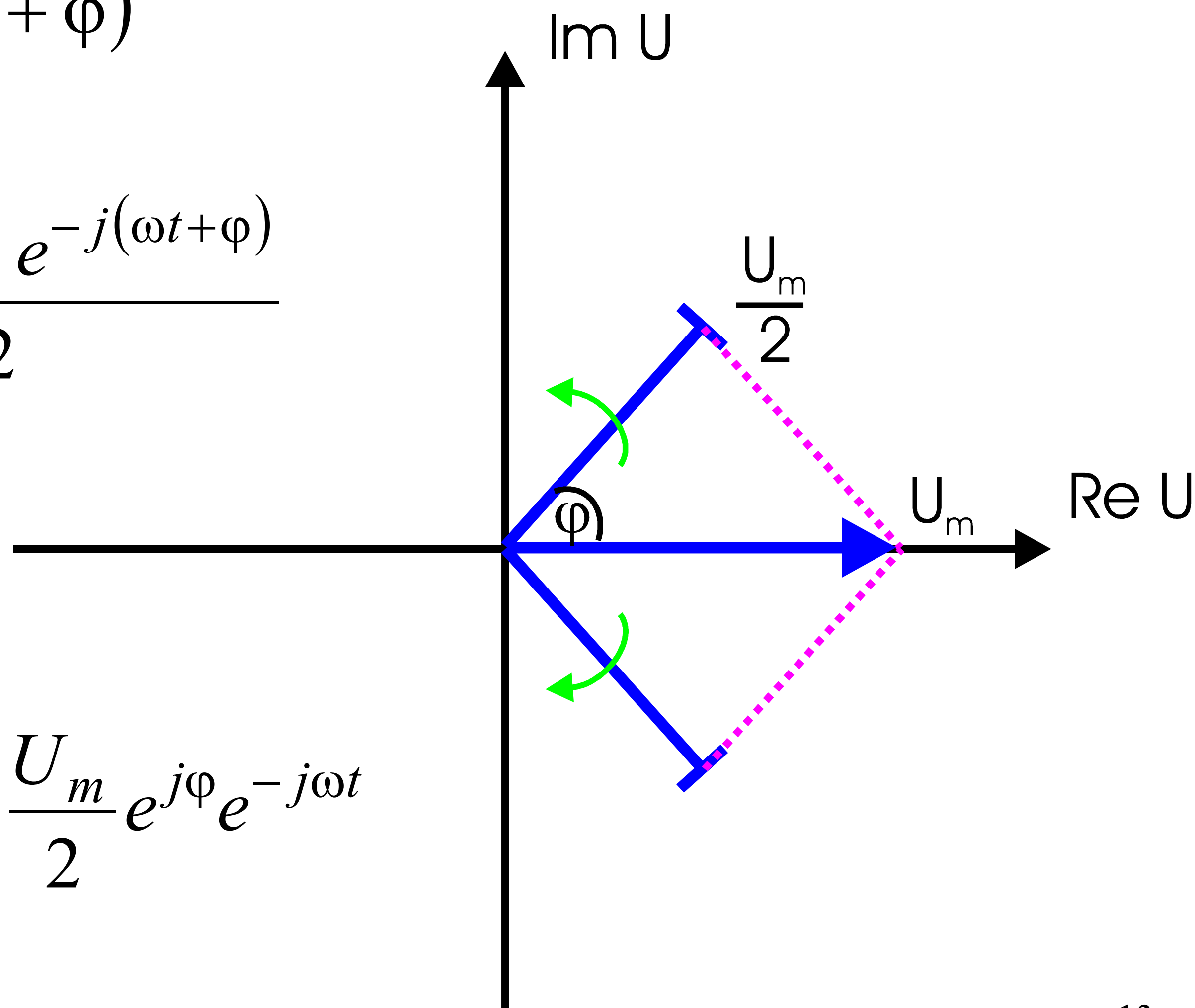
$$\frac{i_1}{i_2} = \frac{-n_2}{n_1} = -\frac{1}{p}$$

Przebiegi harmoniczne

$$U = U_m\, cos(\omega t + \varphi)$$

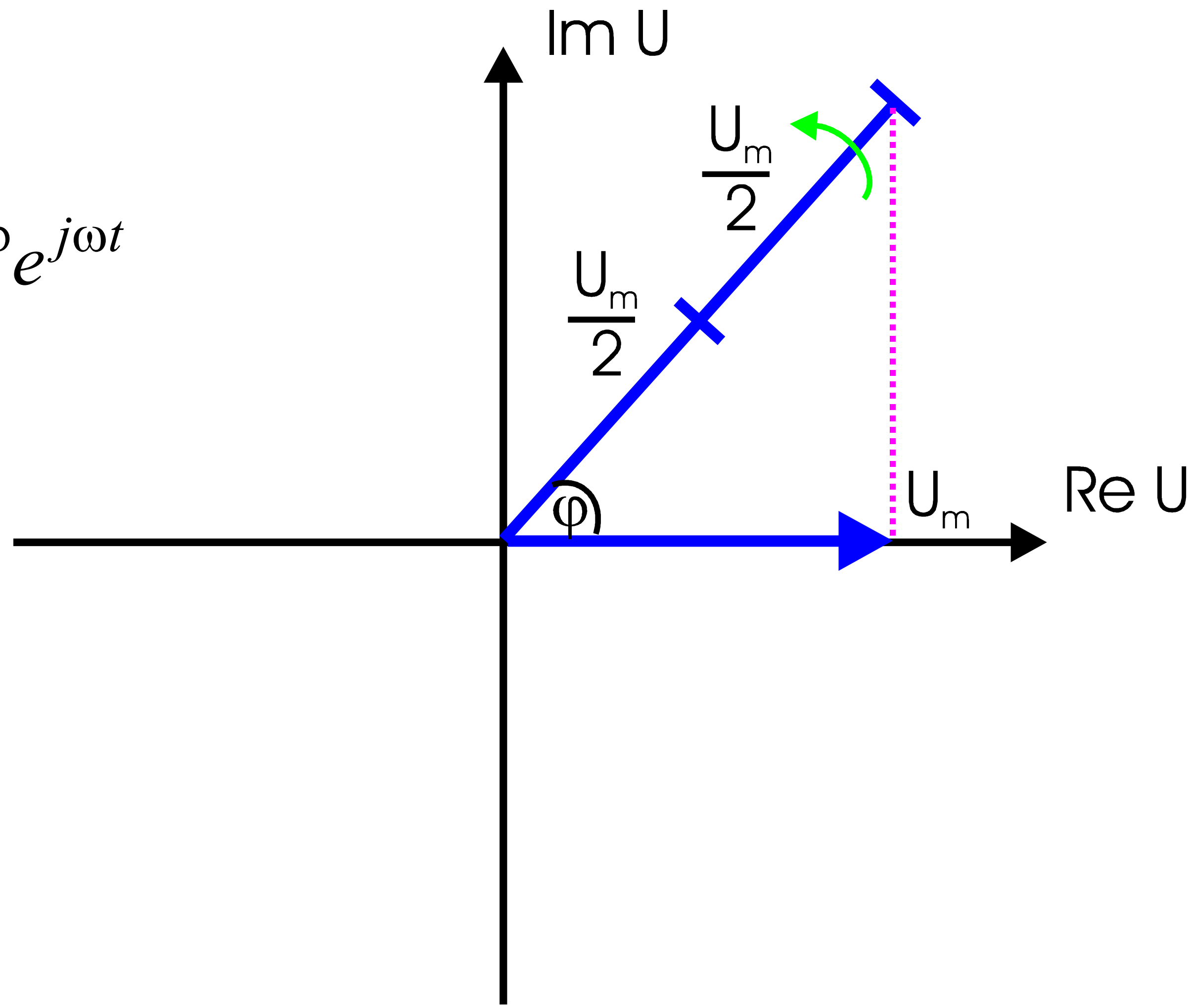
$$U = U_m \frac{e^{j(\omega t + \varphi)} + e^{-j(\omega t + \varphi)}}{2}$$

$$U = \frac{U_m}{2} e^{j\varphi} e^{j\omega t} + \frac{U_m}{2} e^{j\varphi} e^{-j\omega t}$$



Przebiegi harmoniczne

$$U = ReU_m e^{j\varphi} e^{j\omega t}$$



Przebiegi harmoniczne

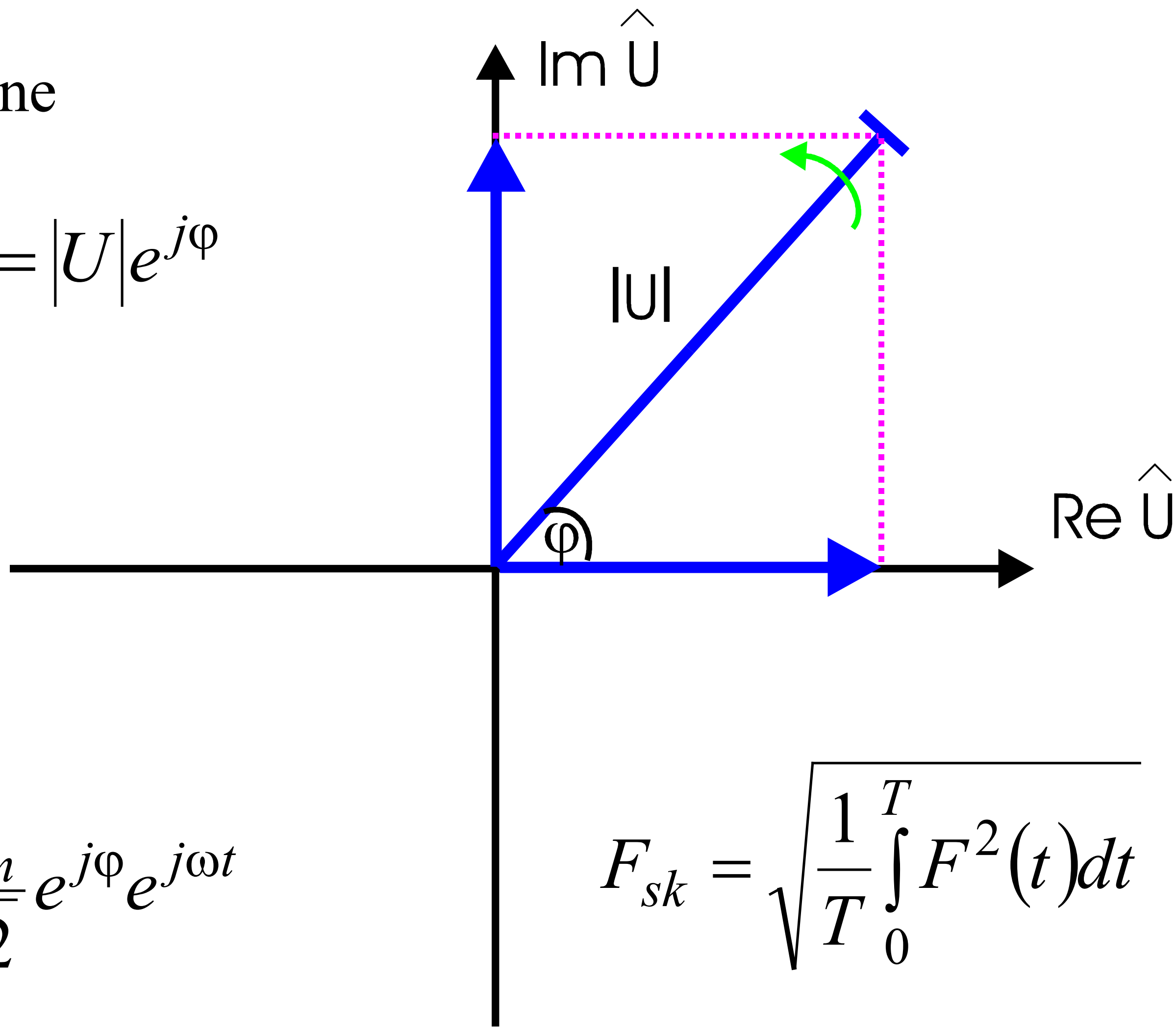
- Napięcie symboliczne

$$\widehat{U} = Re\widehat{U} + j\,Im\widehat{U} = |U|e^{j\varphi}$$

$$|U| = U_{sk} = \frac{U_m}{\sqrt{2}}$$

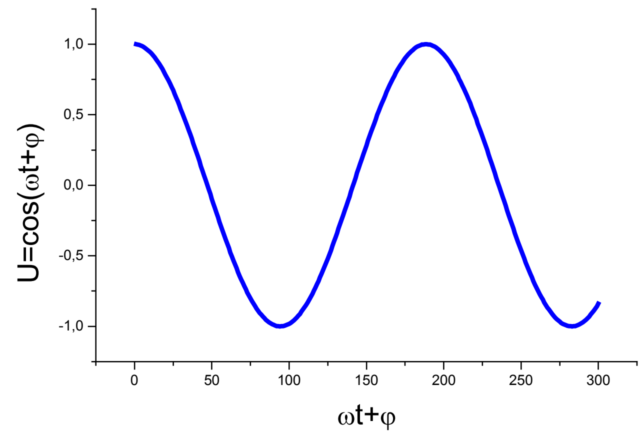
- Wskaz napięcia

$$\boldsymbol{U} = \widehat{U}e^{j\omega t} = \frac{U_m}{\sqrt{2}}e^{j\varphi}e^{j\omega t}$$



$$F_{sk} = \sqrt{\frac{1}{T}\int_0^T F^2(t)\,dt}$$

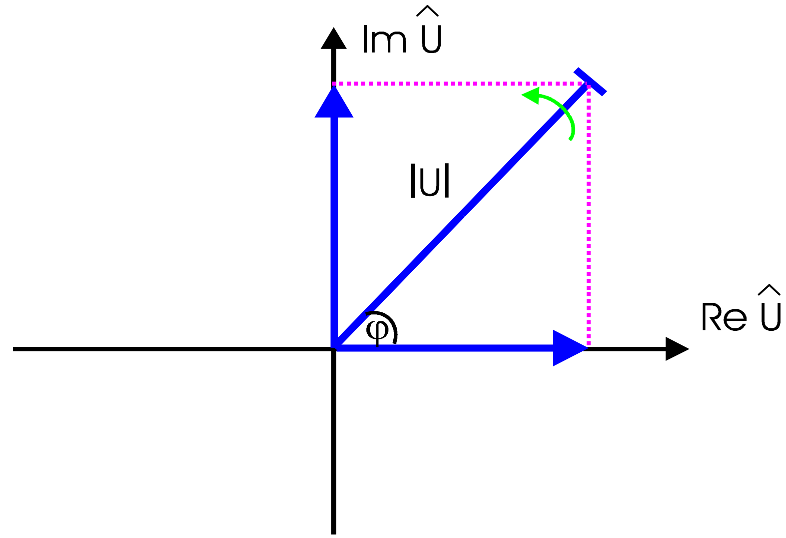
Przebiegi harmoniczne



$$U = U_m \cos(\omega t + \varphi)$$

$$U_m = |U|\sqrt{2}$$

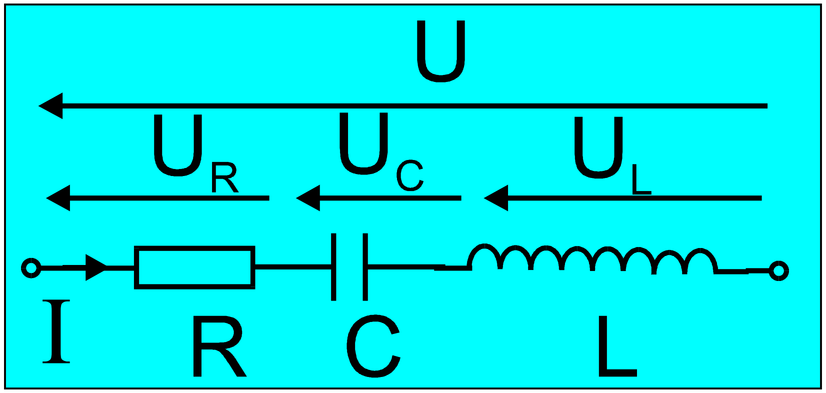
$$|U| = \frac{U_m}{\sqrt{2}}$$



$$\hat{U} = Re\hat{U} + j\,Im\hat{U} = |U|e^{j\varphi}$$

$$U = |U|\sqrt{2}\cos(\omega t + \varphi)$$

Impedancja



$$\hat{U}e^{j\omega t} = \hat{U}_R e^{j\omega t} + \hat{U}_C e^{j\omega t} + \hat{U}_L e^{j\omega t}$$

$$I = \hat{I}e^{j\omega t} \qquad U_L = L\frac{dI}{dt}$$

$$\boldsymbol{U} = \hat{U}e^{j\omega t} = \hat{I}\,Re^{j\omega t} + \frac{\hat{I}}{j\omega C}e^{j\omega t} + \hat{I}Lj\omega e^{j\omega t} \qquad U_C = \frac{1}{C}\int Idt$$

$$Z = \frac{\boldsymbol{U}}{\boldsymbol{I}} = \frac{\hat{U}}{\hat{I}} = |Z|e^{j(\varphi-\nu)} = |Z|e^{j\psi}$$

$$\boxed{Z = R + \frac{1}{j\omega C} + Lj\omega}$$

Impedancja

$$\boxed{Z = R + jX}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Admitancja

$$Y = \frac{1}{Z} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} + j\frac{\left(\frac{1}{\omega C} - \omega L\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\boxed{Y = G + jB}$$

Immitancje

$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{-X}{R^2 + X^2}$$

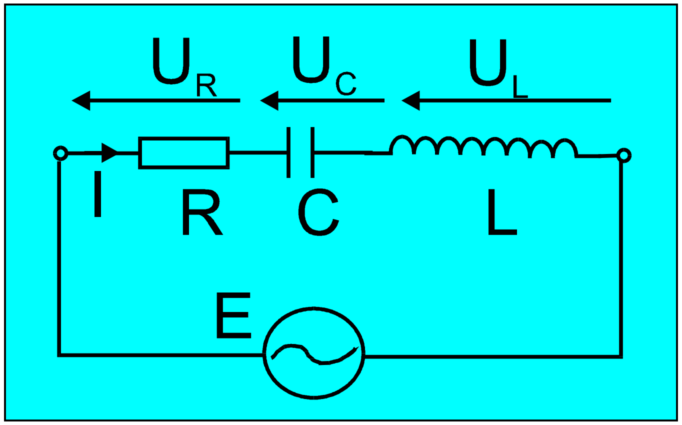
$$Z = R + jX$$

$$Y = G + jB$$

$$R = \frac{G}{G^2 + B^2}$$

$$X = \frac{-B}{G^2 + B^2}$$

Dobroć obwodu



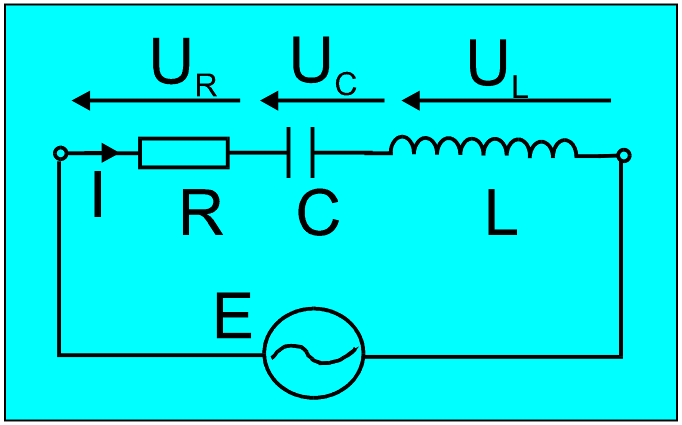
$$Q = 2\pi \frac{maks\, W_{LC}}{W_R}\Bigg|_{\omega=\omega_0,\, t=T}$$

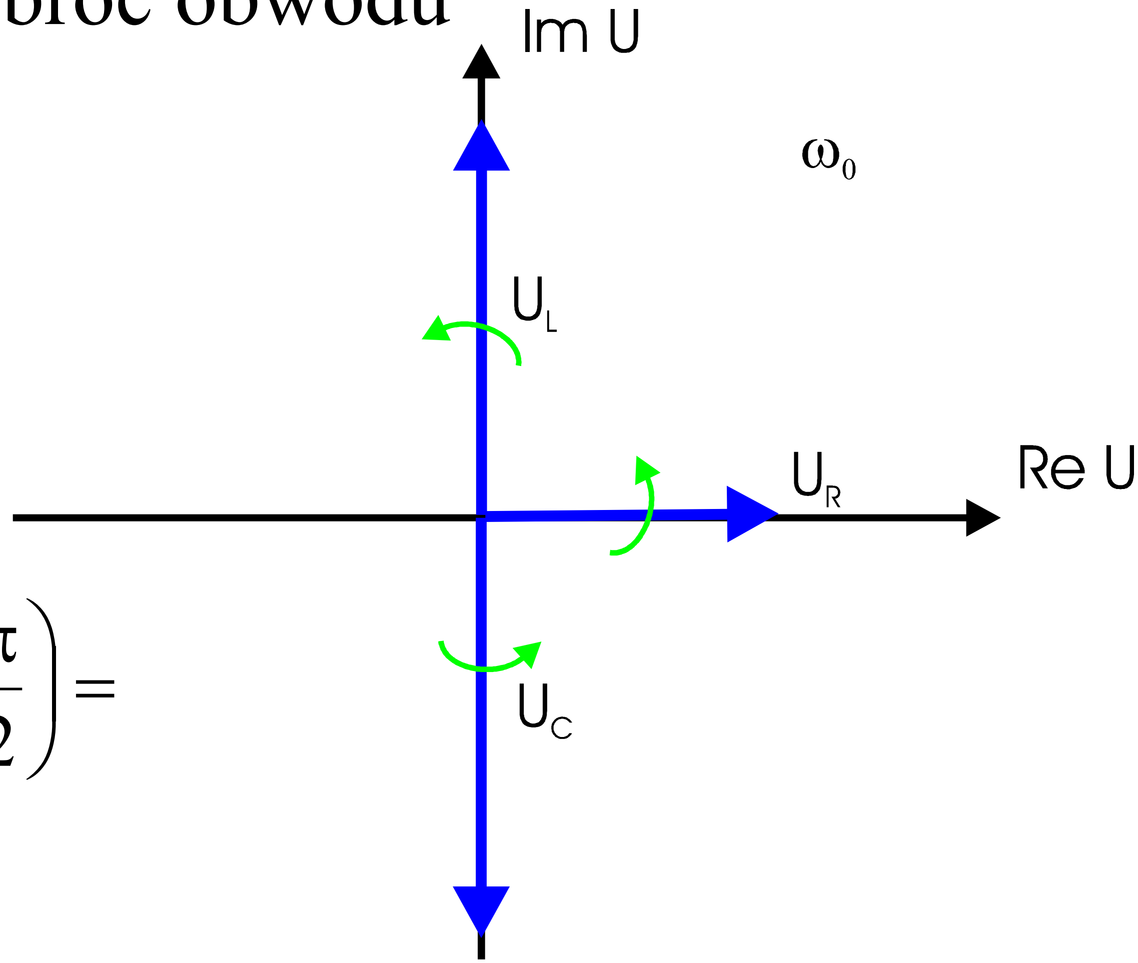
$$W_{LC} = W_L + W_C = \frac{L I_r^2}{2} + \frac{C U_C^2}{2}$$

$$Z\Big|_{\omega=\omega_0 = \frac{1}{\sqrt{LC}}} = R \qquad E = E_m \cos\omega_0 t \qquad I_r = \frac{E_m}{R}\cos\omega_0 t$$

$$\widehat{U} e^{j\omega_0 t} = \widehat{I}_r R e^{j\omega_0 t} + \frac{\widehat{I}_r}{\omega_0 C} e^{j\omega_0 t - j\frac{\pi}{2}} + \widehat{I}_r L\omega_0 e^{j\omega_0 t + j\frac{\pi}{2}}$$

Dobroć obwodu

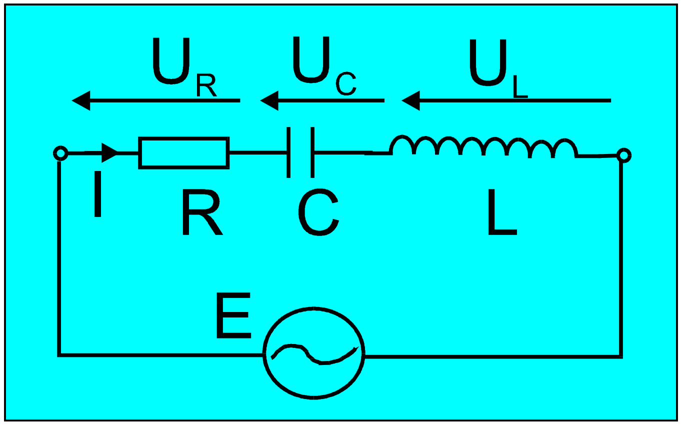




$$U_C = \frac{E_m}{\omega_0 CR} cos\left(\omega_0 t - \frac{\pi}{2}\right) =$$

$$\frac{E_m}{\omega_0 CR} sin\,\omega_0 t$$

Dobroć obwodu



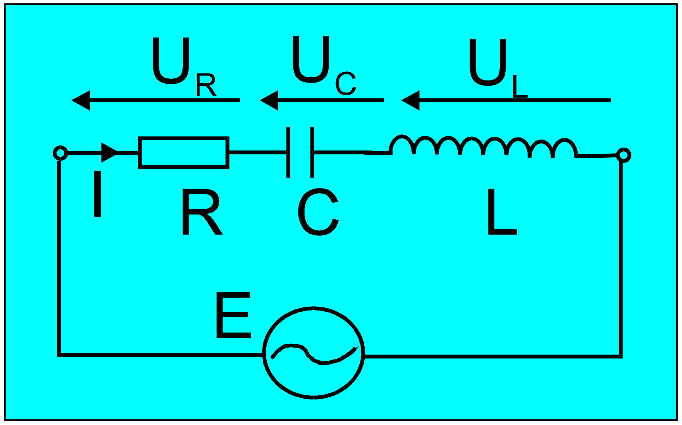
$$W_R = I_{sk}^2 RT = \frac{E_m^2}{2R} T \Big|_{T = 2\pi / \omega_0} = \frac{1}{2} E_m^2 \frac{2\pi}{\omega_0 R}$$

$$W_{LC} = W_L + W_C = \frac{LE_m^2}{2R} cos^2\, \omega_0 t + \frac{E_m^2}{2\omega_0^2 CR^2} sin^2\, \omega_0 t$$

$$W_{LC} = \frac{LE_m^2}{2} = \frac{1}{2} E_m^2 \frac{\sqrt{\frac{L}{C}}}{\omega_0 R^2} = \frac{1}{2} E_m^2 \frac{\rho}{\omega_0 R^2}$$

Opór charakterystyczny

Dobroć obwodu

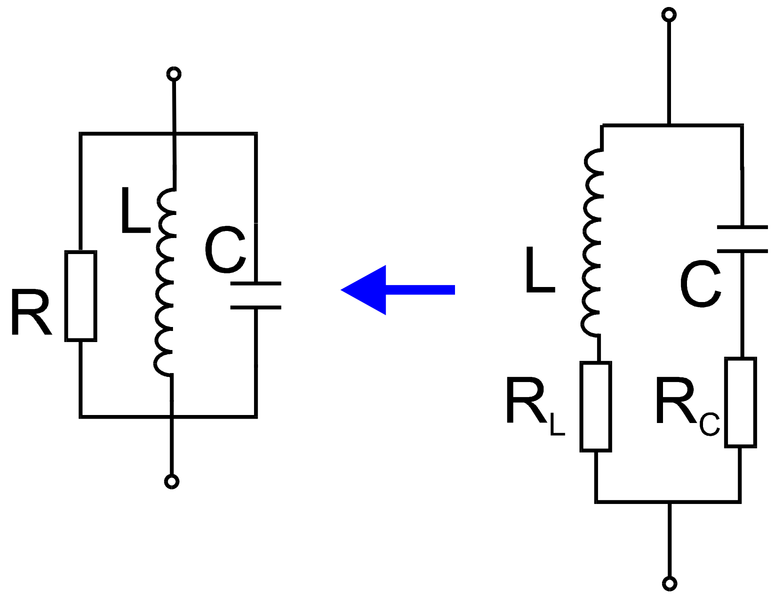


$$Q = 2\pi \frac{\frac{1}{2} E_m^2 \frac{\rho}{\omega_0 R^2}}{\frac{1}{2} E_m^2 \frac{2\pi}{\omega_0 R}} = \frac{\rho}{R}$$

$$\boxed{Q = \frac{L\omega_0}{R} = \frac{1}{\omega_0 RC}}$$

$$\widehat{U} e^{j\omega_0 t} = \widehat{I}_r R e^{j\omega_0 t} + \widehat{I}_r R Q e^{j\omega_0 t - j\frac{\pi}{2}} + \widehat{I}_r R Q e^{j\omega_0 t + j\frac{\pi}{2}}$$

Dobroć obwodu równoległego



$$Q = \frac{R}{L\omega_0} = \omega_0 RC$$

$$R_d\Big|_{\omega \approx \omega_0} = \frac{L}{C\left(R_L + R_C\right)}$$

Opór dynamiczny - liczony przy małych wartościach oporności pasożytniczych !!!

Zależności energetyczne

- cewka

$$W_L = \frac{LI_L^2}{2} \qquad P_L = \frac{L}{2}\frac{dI_L^2}{dt} = \frac{L}{2}\frac{d\left(\widehat{I}_L^2 e^{j2\omega t}\right)}{dt} = j\omega L\, \widehat{I}_L^2 e^{j2\omega t}$$

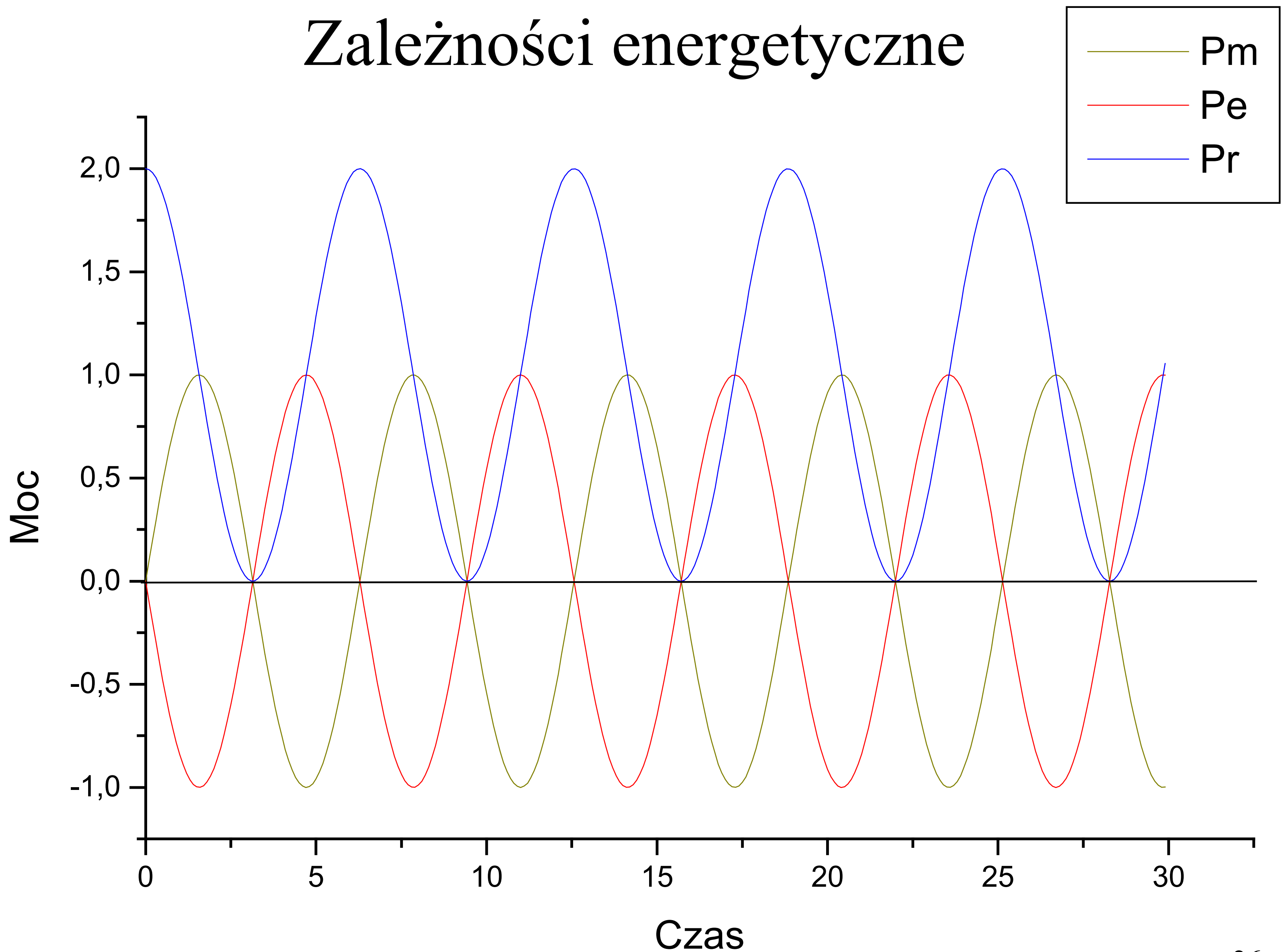
- kondensator

$$W_C = \frac{CU_C^2}{2} \qquad P_C = \frac{C}{2}\frac{dU_C^2}{dt} = \frac{C}{2}\frac{d\left(\widehat{U}_C^2 e^{j2\omega t}\right)}{dt} = \frac{1}{j\omega C}\, \widehat{I}_C^2 e^{j2\omega t}$$

- rezystor

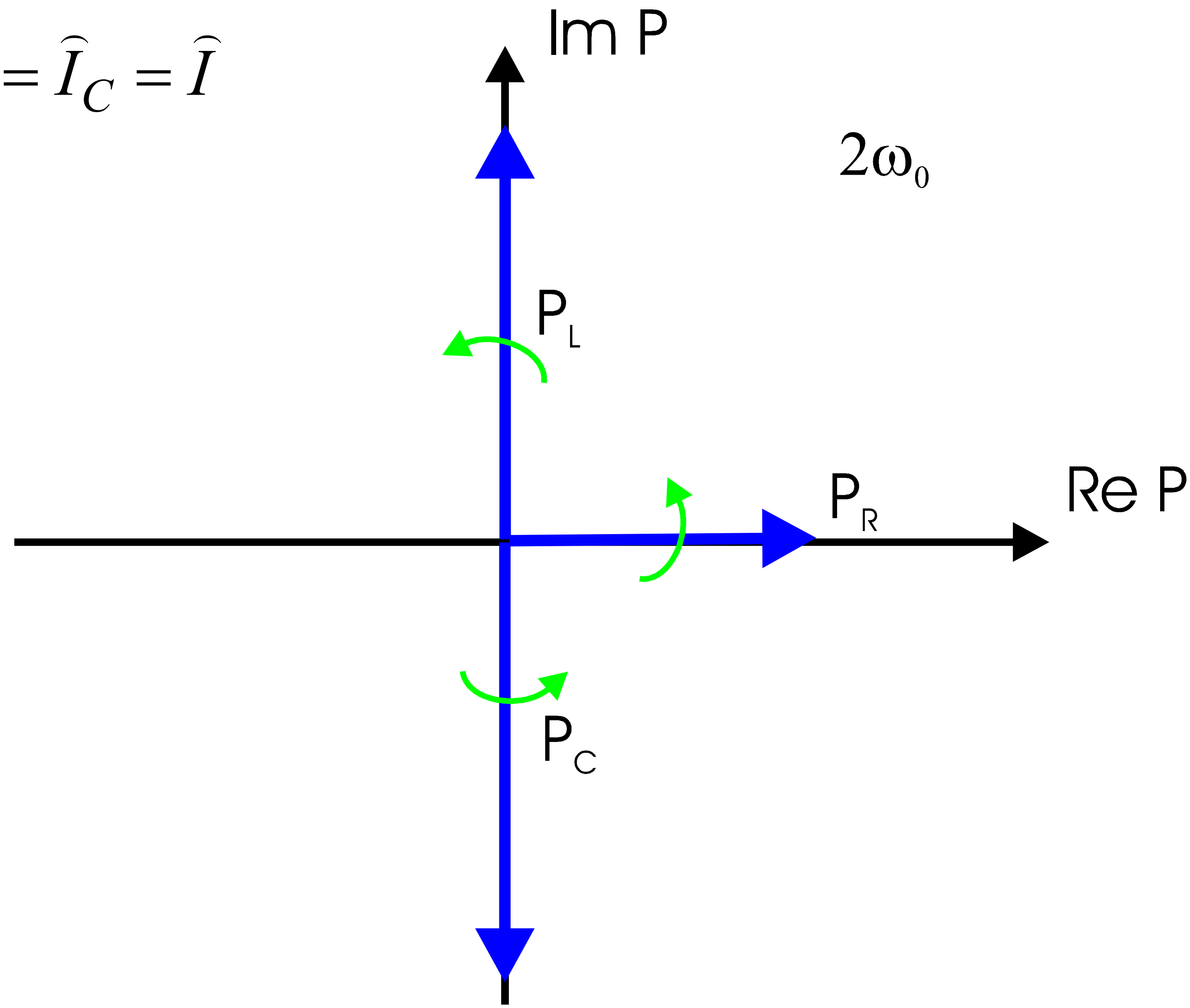
$$P_R = I_R U_R = \widehat{I}_R \widehat{U}_R e^{j2\omega t} = R\widehat{I}_R^2 e^{j2\omega t}$$

Zależności energetyczne

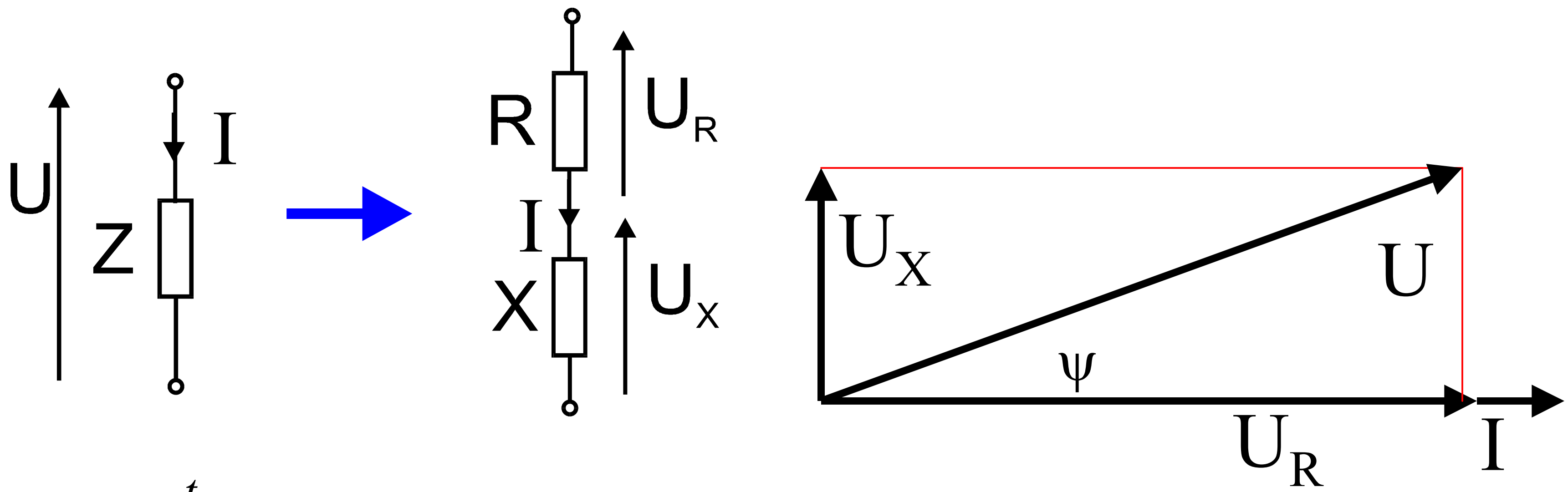


Zależności energetyczne

$$\widehat{I}_R = \widehat{I}_L = \widehat{I}_C = \widehat{I}$$



Zależności energetyczne

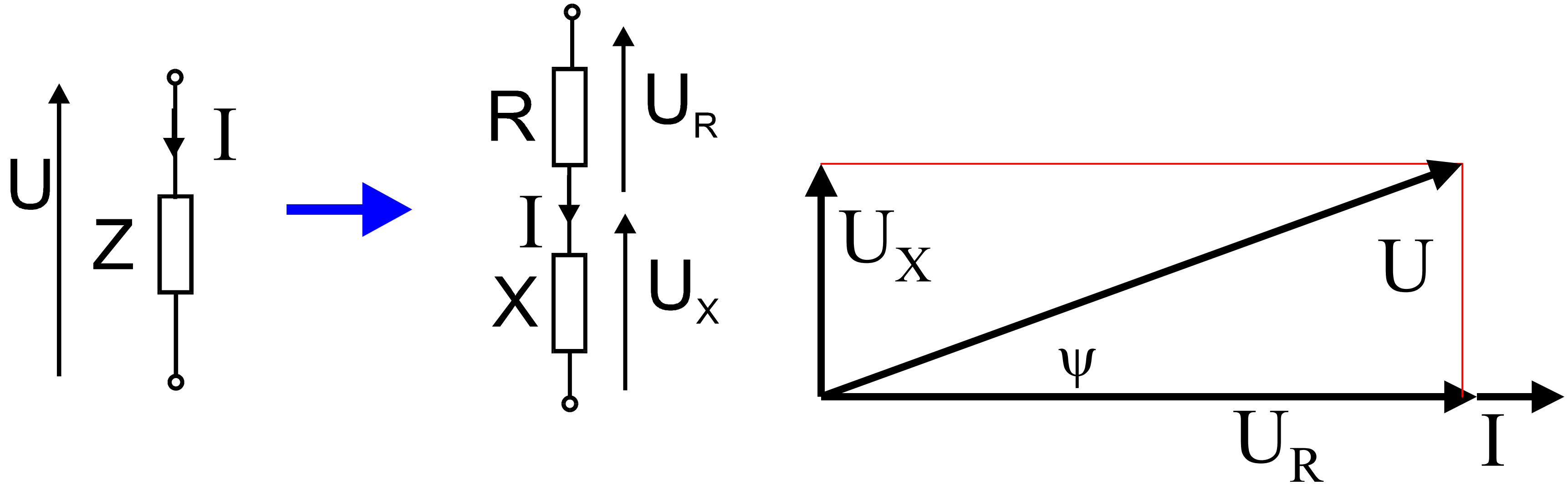


$$P = \frac{1}{T}\int_{t_1}^{t_2} U_m \cos(\omega t + \varphi) I_m \cos(\omega t + \varphi - \psi) dt$$

$P = |U_R I| = |UI| \cos\psi$ - moc czynna

$Q = |U_X I| = |UI| \sin\psi$ - moc bierna

Zależności energetyczne



- moc zespolona

$$P_z = P + jQ = |UI| \cos\psi + j|UI| \sin\psi = |UI| e^{j\psi}\left(e^{j\varphi} e^{-j\varphi}\right) =$$

$$|U| e^{j\varphi} |I| e^{-j(\varphi-\psi)} = UI^*$$

$$P_s = |P_z| = \sqrt{P^2 + Q^2}$$ - moc pozorna

Moc przy przebiegach odkształconych

$$U = U_{m_0} + \sum_{k=1}^{\infty} U_{m_k} \, sin(k\omega t + \varphi_k)$$

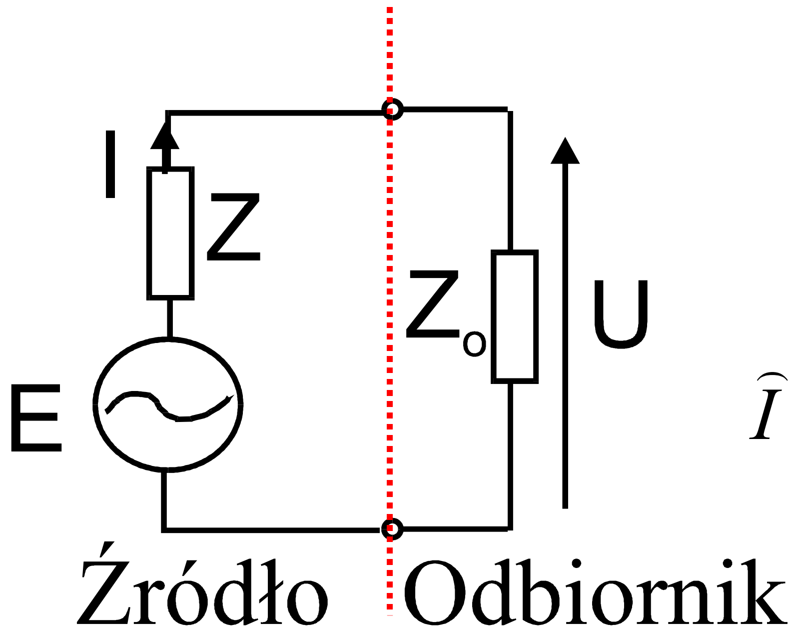
$$I = I_{m_0} + \sum_{l=1}^{\infty} I_{m_l} \, sin(l\omega t + \nu_l)$$

$$P = \frac{1}{T} \int_0^T \left[\sum_{k=0}^{\infty} U_{m_k} \, sin(k\omega t + \varphi_k) \right] \left[\sum_{l=0}^{\infty} I_{m_l} \, sin(l\omega t + \nu_l) \right] dt$$

$$P = \frac{1}{T} \sum_{k=0}^{\infty} \int_0^T U_{m_k} \, sin(k\omega t + \varphi_k) I_{m_k} \, sin(k\omega t + \nu_k) dt$$

$$P = U_0 I_0 + \sum_{k=1}^{\infty} \left| U_k I_k \right| cos \, \psi_k$$

Dopasowanie ze względu na moc czynną



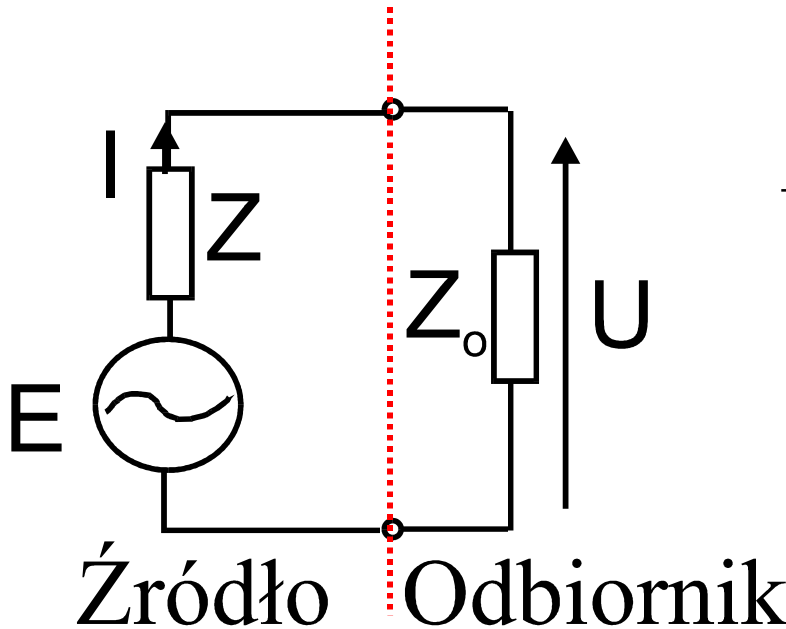
$$Z = R + jX$$

$$Z_o = R_o + jX_o$$

$$\widehat{I} = \frac{\widehat{E}}{Z + Z_o} = \frac{\widehat{E}}{R + R_o + j\left(X + X_o\right)}$$

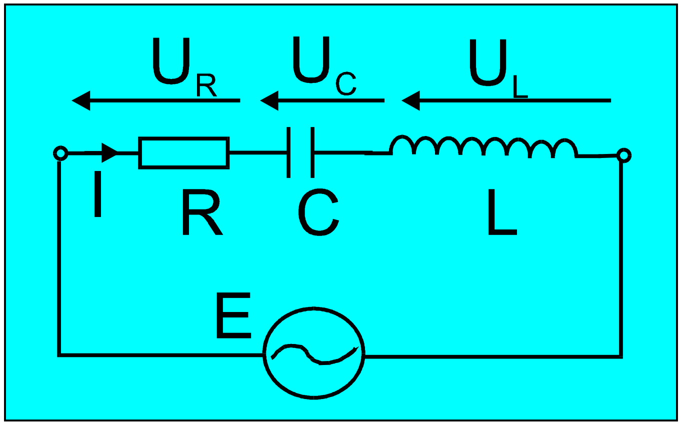
$$\widehat{P} = \left|\widehat{U}_{R_o}\widehat{I}\right| = \left|\widehat{I}^2\right| R_o = \frac{\widehat{E}^2 R_o}{\left(R_o + R\right)^2 + \left(X_o + X\right)^2}$$

Dopasowanie ze względu na moc czynną



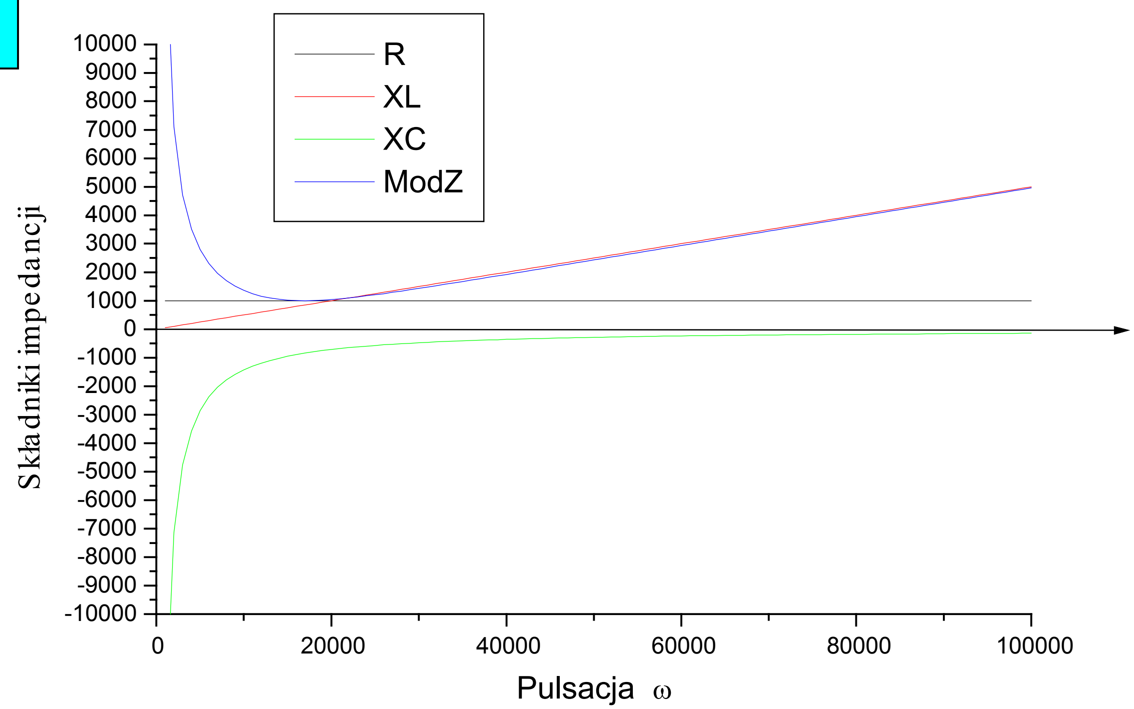
$$\left.\begin{aligned} \frac{\partial P}{\partial X_o} &= 0 \\ \frac{\partial P}{\partial R_o} &= 0 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} X_o &= -X \\ R_o &= R \end{aligned}}$$

Obwód rezonansowy - szeregowy

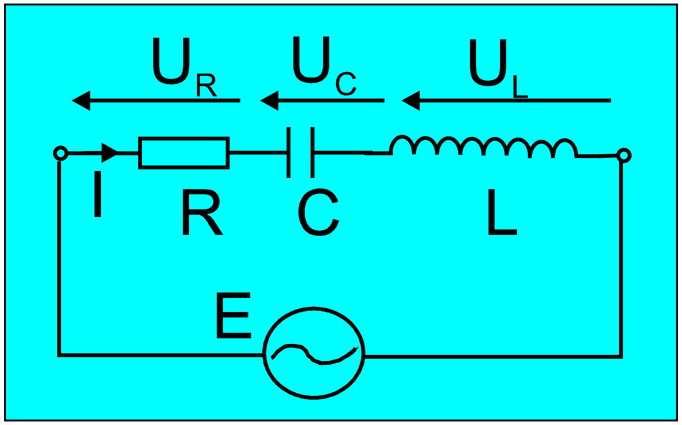


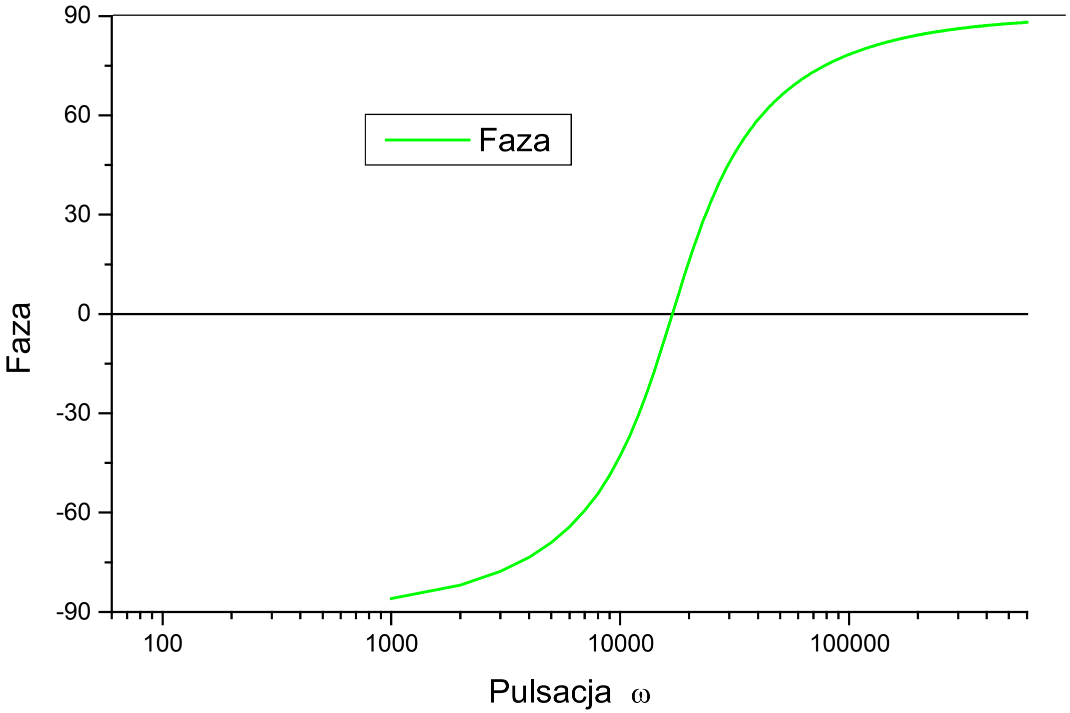
$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Obwód rezonansowy - szeregowy





$$Y = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

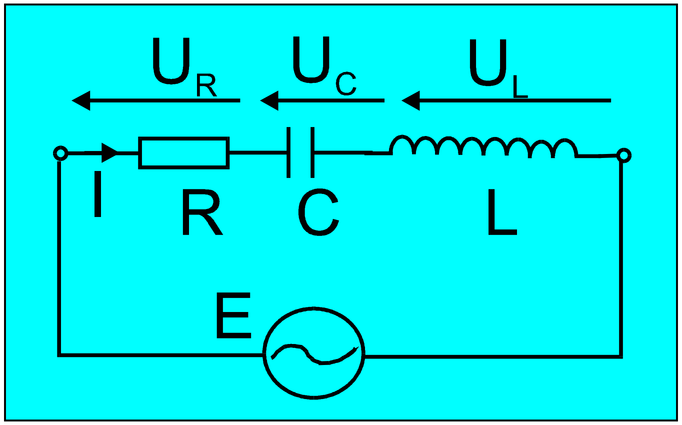
$$\gamma = \frac{\omega}{\omega_0}$$

Pulsacja względna

Dobroć obwodu

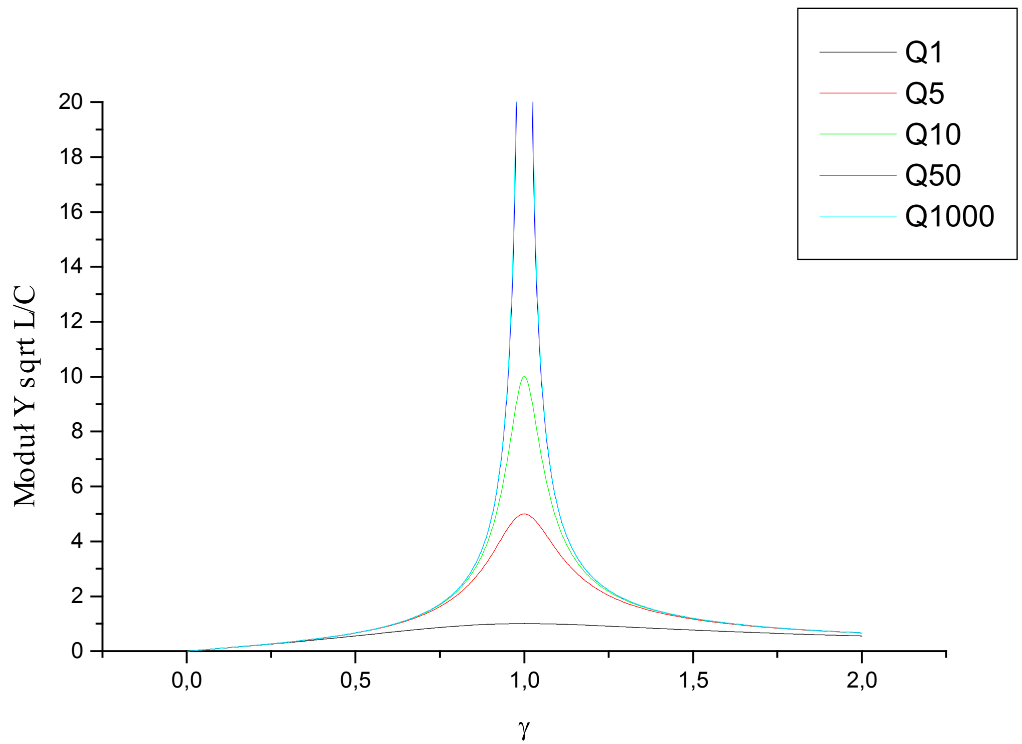
$$Q = \frac{\omega_0 L}{R}$$

Obwód rezonansowy - szeregowy



$$Y = \sqrt{\frac{C}{L}} \frac{Q}{1 + jQ\left(\gamma - \frac{1}{\gamma}\right)}$$

$$|Y|\sqrt{\frac{L}{C}} = \frac{1}{\sqrt{\frac{1}{Q^2} + \left(\gamma - \frac{1}{\gamma}\right)^2}}$$



Obwód rezonansowy - szeregowy

Rozstrojenie bezwzględne

$$\xi = \frac{X}{R}$$

Rozstrojenie względne

$$\nu = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

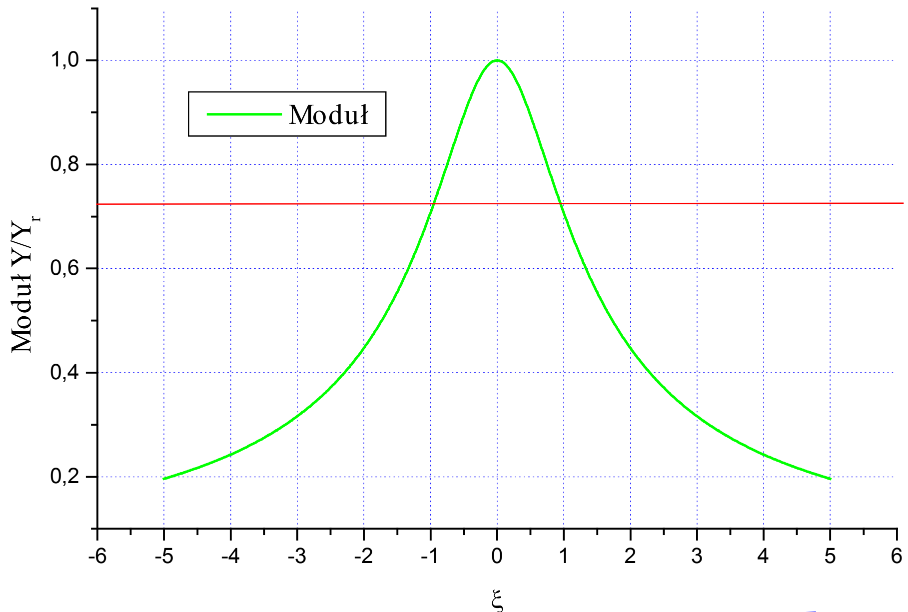
$$Y = \frac{\frac{1}{R}}{1 + j\xi} = |Y| e^{-j\varphi} = \frac{\frac{1}{R}}{\sqrt{1+\xi^2}} e^{-j\,ar\,ctg\,\xi}$$

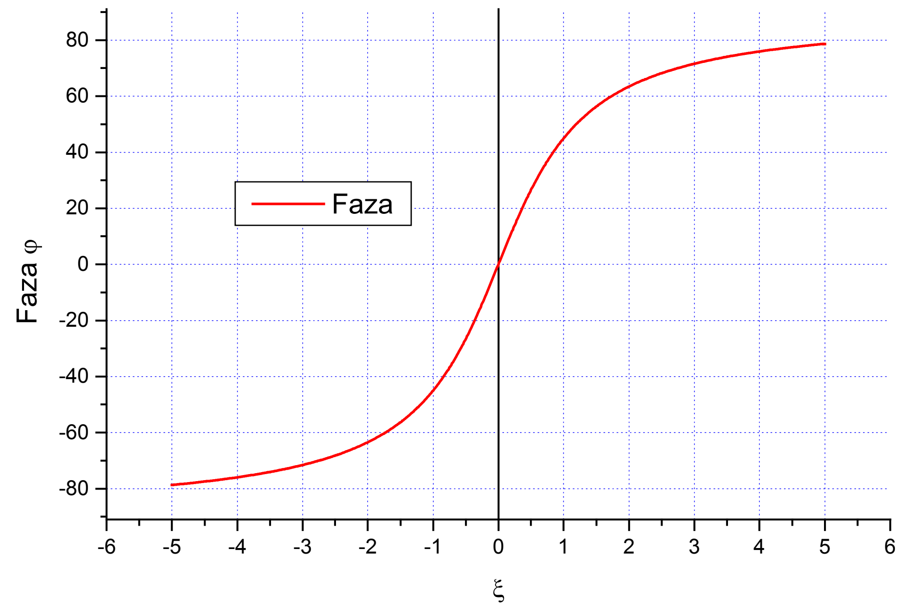
$$\boxed{\frac{|Y|}{|Y_r|} = \frac{1}{\sqrt{1+\xi^2}}}$$

$$\varphi = ar\,ctg\,\xi$$

$$\xi = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{\omega_0 L \frac{\omega}{\omega_0}}{R} - \frac{\frac{1}{\omega_0 C}\frac{\omega_0}{\omega}}{R} = Q\nu$$

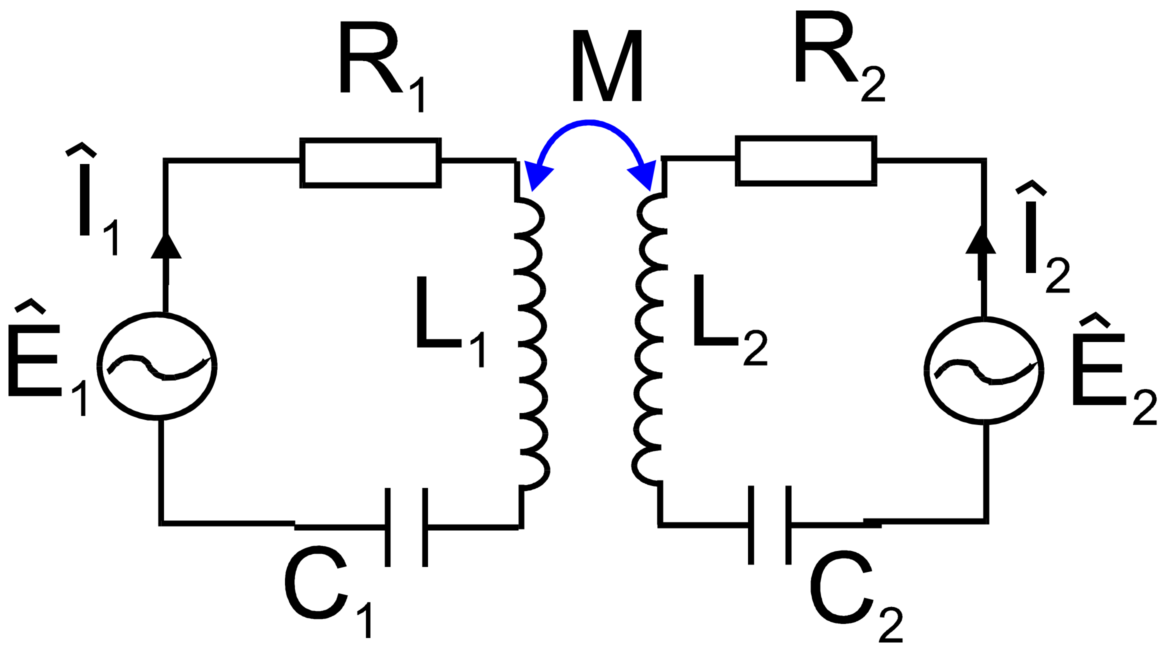
Obwód rezonansowy - szeregowy





Uniwersalna krzywa rezonansowa

Obwody sprzężone



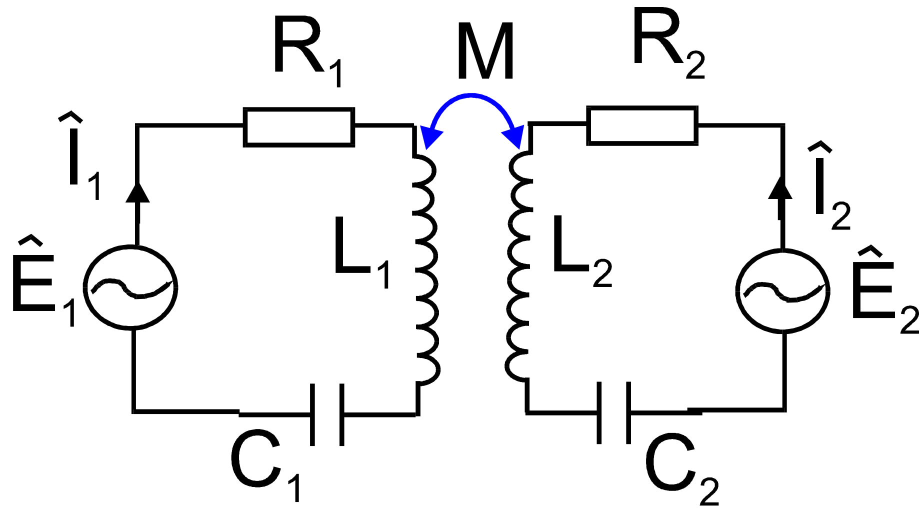
$$Z_1 = R_1 + j\omega L_1 + \frac{1}{j\omega C_1}$$

$$Z_2 = R_2 + j\omega L_2 + \frac{1}{j\omega C_2}$$

$$R_1\hat{I}_1 + j\omega L_1\hat{I}_1 + \frac{1}{j\omega C_1}\hat{I}_1 + j\omega M\hat{I}_2 = \hat{E}_1$$

$$R_2\hat{I}_2 + j\omega L_2\hat{I}_2 + \frac{1}{j\omega C_2}\hat{I}_2 + j\omega M\hat{I}_1 = \hat{E}_2$$

Obwody sprzężone

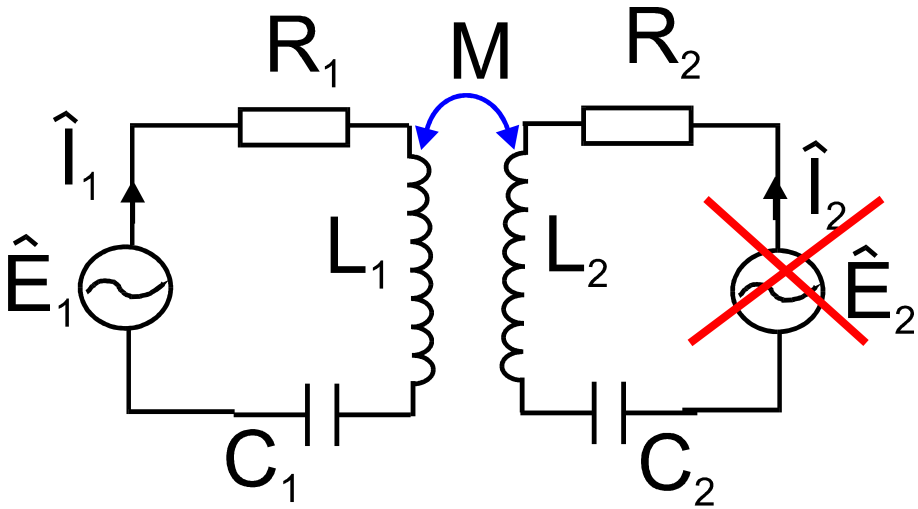


$$\left.\begin{aligned} Z_1\hat{I}_1 + j\omega M\hat{I}_2 &= \hat{E}_1 \\ j\omega M\hat{I}_1 + Z_2\hat{I}_2 &= \hat{E}_2 \end{aligned}\right\}$$

$$\left.\begin{aligned} \hat{I}_1 &= \frac{\hat{E}_1 - j\omega M\hat{I}_2}{Z_1} \\ \hat{I}_2 &= \frac{\hat{E}_2 - j\omega M\hat{I}_1}{Z_2} \end{aligned}\right\}$$

$$\left.\begin{aligned} \hat{I}_1 &= \frac{\hat{E}_1 Z_2 - j\omega M\hat{E}_2}{Z_1 Z_2 + \omega^2 M^2} \\ \hat{I}_2 &= \frac{\hat{E}_2 Z_1 - j\omega M\hat{E}_1}{Z_1 Z_2 + \omega^2 M^2} \end{aligned}\right\}$$

Obwody sprzężone



Źródło tylko po stronie pierwotnej

$$\left.\begin{aligned} \hat{I}_1 &= \frac{\hat{E}_1 Z_2}{Z_1 Z_2 + \omega^2 M^2} \\ \hat{I}_2 &= \frac{-j\omega M \hat{E}_1}{Z_1 Z_2 + \omega^2 M^2} \end{aligned}\right\}$$

$$\left.\begin{aligned} \hat{I}_1 &= \frac{\hat{E}_1}{Z_1 + \dfrac{\omega^2 M^2}{Z_2}} \\ \hat{I}_2 &= \frac{-\hat{E}_1}{Z_1} \frac{j\omega M}{Z_2 + \dfrac{\omega^2 M^2}{Z_1}} \end{aligned}\right\}$$

Impedancja przeniesiona z obwodu wtórnego do pierwotnego

Impedancja przeniesiona z obwodu pierwotnego do wtórnego

Obwody sprzężone

Admitancja wzajemna

$$Y_{12} = \frac{-\hat{I}_2}{\hat{E}_1} = \frac{j\omega M}{Z_1 Z_2 + \omega^2 M^2}$$

Oznaczamy rozstrojenia bezwzględne: $\xi_1 = \frac{X_1}{R_1}$ $\xi_2 = \frac{X_2}{R_2}$

Oznaczamy wskaźnik sprzężenia

$$A = \frac{\omega M}{\sqrt{R_1 R_2}}$$

$$\boxed{Y_{12} = \frac{j2A\left|Y_{12}\right|_{mm}}{1 + A^2 - \xi_1\xi_2 + j\left(\xi_1 + \xi_2\right)}}$$

Gdzie: $\left|Y_{12}\right|_{mm} = \frac{1}{2\sqrt{R_1 R_2}}$

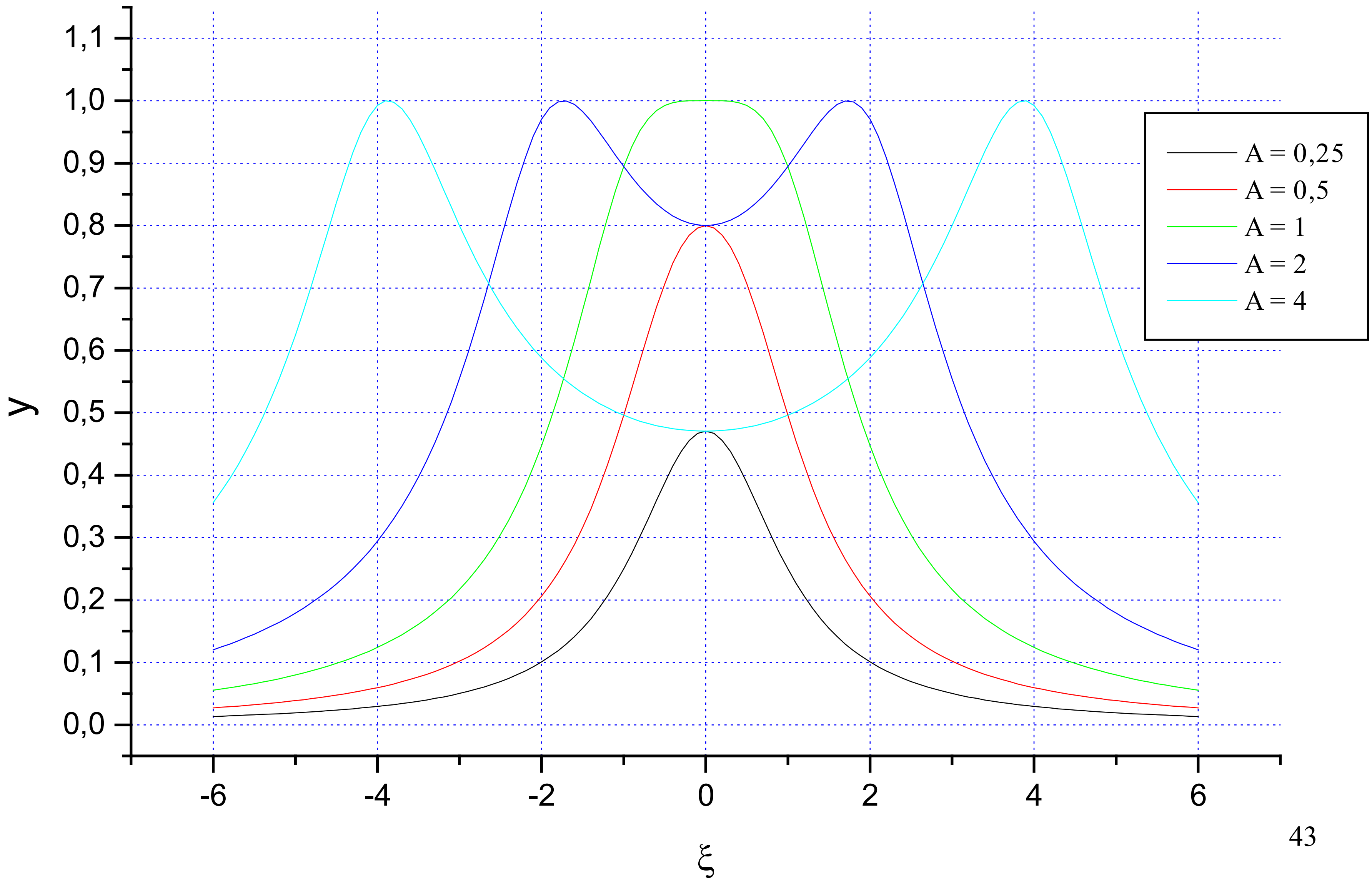
Jednakowe obwody sprzężone

$Q_1 = Q_2 = Q;\ \ \xi_1 = \xi_2 = \xi\ ;\ \omega_{01} = \omega_{02} = \omega_0\ ;$

$$y = \frac{|Y_{12}|}{|Y_{12}|_{mm}} = \frac{2A}{\sqrt{\left(1 + A^2 - \xi^2\right)^2 + 4\xi^2}}$$

$$\varphi_{12} = ar\,ctg\,\frac{2\xi}{1 + A^2 - \xi^2}$$

Jednakowe obwody sprzężone



Jednakowe obwody sprzężone