Applications of Computers in Education. List No. 2

1. Prepare an animation illustrating glide reflection and the commutativity of the symmetry and translation composing it.
2. Through one of the two intersection points of two circles draw a line from which the circles cut chords of equal length.
3. Four lines a_1, a_2, a_3, a_4 are given, no two of which are parallel. Construct a parallelogram $A_1A_2A_3A_4$ such that $A_i \in a_i$ for all i.
4. Construct triangle ABC given its vertices A, B and the line l of the angle bisector at vertex C.
5. A chord CD of a given circle intersects the diameter AB at point M at an angle of $45°$. Show that the sum $|CM|^2 + |DM|^2$ does not depend on the choice of point M.
6. Circles ω_1 and ω_2 with equal radii are internally tangent to a larger circle ω at points A_1 and A_2, respectively. For any point $C \in \omega$ the segments A_1C and A_2C intersect ω_1 and ω_2 at points B_1 and B_2, respectively. Show that $A_1A_2 \parallel B_1B_2$.
7. Points A, B lie on the same side of the line $a = l(M, N)$. Determine a point $X \in a$ for which $|\angle AXM| = 2|\angle BXN|$.
8. A circle ω and lines a, b, c are given, no two of which are parallel. Inscribe in ω a triangle whose consecutive sides are parallel respectively to a, b and c.
9. Points A, B lie on the same side of line l. Place on line l a segment MN of given length d such that:
   (a) the broken line $AMNB$ is the shortest;
   (b) the equality $|AM| = |NB|$ holds.
10. Two circles ω_1, ω_2 and a line l are given. Construct:
    (a) A line l_1 parallel to l intersecting the circles at points with a given distance a.
    (b) A line l', on which the circles cut chords of the same length.
11. Points M and K belong respectively to sides BC and CD of the square $ABCD$ such that $\angle BAM \equiv \angle MAK$. Show that $|BM| + |KD| = |AK|$.
12. Squares $ABCD$ and $BKMN$ with common vertex B are described according to the positive orientation of the plane. Show that the median BE of triangle ABK and the altitude BF of triangle CBN lie on the same line.
13. In the regular hexagon $ABCDEF$, point G is the midpoint of diagonal BD, and point H is the midpoint of side EF. Show that triangle AGH is equilateral.
14. Point M lies on arc AB of the circumcircle of equilateral triangle ABC. Show that $|MC| = |MA| + |MB|$.
15. For a given triangle ABC find a point D such that the sum of lengths $|AD| + |BD| + |CD|$ is minimal.
16. Prepare animations illustrating spiral similarity and dilative symmetry, as well as the commutativity of the homotheties and isometries composing them.
17. Point P lies inside a given angle. Construct a circle that contains P and is tangent to the sides of the angle.
18. Points A, B and line l are given. Determine the locus of centroids of triangles ABC whose vertex C lies on line l.
19. Inscribe in a given triangle ABC a triangle homothetic to a given triangle DEF.
20. A point P lies on side AB of triangle ABC. Inscribe in triangle ABC a triangle PXY similar to a given triangle DEF (use spiral similarity).