

Parametrization

1. Find parametric equation for the line parallel to the vector $[2, 3, 4]$ and through the point $(1, 5, 7)$.
2. Find the points at which the line $x = t, y = 2t, z = 1 + t$ pierces the sphere of radius 10 centered at the origin.
3. Two particles move through space, with equations $\vec{r}_1(t) = [t, 1 + 2t, 3 - 2t]$ and $\vec{r}_2(t) = [-2 - 2t, 1 - 2t, 1 + t]$. Do the particles ever collide? Do their paths cross?
4. Are the lines $x = -1 + t, y = 1 + 2t, z = 5 - t$ and $x = 2 + 2t, y = 4t, z = 3 + t$ parallel? Do they intersect?
5. Show that the following two lines are the same:

$$\vec{r} = [-1, -1, 1] + t[3, 6, -3] \quad \vec{r} = [1, 3, -1] + t[-1, -2, 1].$$

6. Find parametric equations for:
 - (a) The line in the direction of the vector $[1, 2, -1]$ and through the point $(3, 0, -4)$.
 - (b) The line parallel to the z -axis passing through the point $(1, 0, 0)$.
 - (c) The line through the points $(1, 5, 2)$ and $(5, 0, -1)$.
 - (d) The line intersecting the x -axis at $x = 3$ and the z -axis at $z = -5$.
 - (e) The circle of radius 3 in the xy -plane, centered at the origin, counterclockwise.
 - (f) The circle of radius 3 parallel to the xy -plane, centered at the point $(0, 0, 2)$.
7. Find the velocity and acceleration vectors:
 - (a) $x = t, y = t^3 - t,$
 - (b) $x = 3 \cos(t^2), y = 3 \sin(t^2), z = t^2,$
8. Find the velocity $\vec{v}(t)$ and the speed $|\vec{v}(t)|$. Find any times at which the particle stops.
 - (a) $x = 5t^2, y = t^3 + 1,$
 - (b) $x = 3 \sin(t^2) - 1, y = 3 \cos(t^2).$
9. Find the length of the following curves. Explain your answer.
 - (a) $x = 3 + 5t, y = 1 + 4t, z = 3 - t$ for $1 \leq t \leq 2$.
 - (b) $x = \cos(e^t), y = \sin(e^t)$ for $0 \leq t \leq 1$.
 - (c) $x = \cos 3t, y = \sin 5t$ for $0 \leq t \leq 2\pi$.