

1. Find the partial derivatives

(a) f_x and f_y if $f(x, y) = 5x^2y^3 + 8xy^2 - 3x^2$

(b) $\frac{\partial}{\partial y}(3x^5y^7 - 32x^4y^3 + 5xy)$

(c) $\frac{\partial}{\partial x}(y\sqrt{x})$

(d) F_v if $F = \frac{mv^2}{2}$

(e) $\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{2\pi\sigma}}e^{-(x-\mu)^2/(2\sigma^2)}\right)$

(f) $\frac{\partial m}{\partial v}$ if $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$

(g) g_x if $g(x, y) = \ln(ye^{xy})$

(h) $\frac{\partial z}{\partial y}|_{(1,0,5)}$ if $z = e^{x+2y} \sin y$

(i) $\frac{\partial f}{\partial x}|_{(\pi/3,1)}$ if $f(x, y) = x \ln(y \cos x)$

2. Find the equation for the tangent plane to the following surfaces at given points

(a) the surface $z = \sqrt{9 - x^2 - y^2}$, the point $(x_0, y_0, z_0) = (\sqrt{2}, -\sqrt{3}, 2)$

(b) the surface $z = x^y$, the point $(x_0, y_0, z_0) = (2, 4, 16)$

(c) the surface $z = \frac{\arcsin x}{\arccos y}$, the point $(x_0, y_0, z_0) = (-\frac{1}{2}, \frac{\sqrt{3}}{2}, -1)$

3. Compute the approximation of the following expressions

(a) $\frac{\arctan 0.9}{\sqrt{4.02}}$

(b) $\frac{0.98^{1.01}}{1.01^{2.01}}$

4. Calculate the directional derivative of $f(x, y) = x^2 + y^2$ at $(1, 0)$ in the direction of the vector $\vec{v} = [1, 1]$.

(a) $f(x, y) = x^2 + xy + 3y - 1$ at $(x_0, y_0) = (1, 1)$, in the direction of the vector $\vec{v} = [2, 1]$

(b) $f(x, y) = \sqrt[3]{xy^2}$ at $(x_0, y_0) = (0, 0)$ in the direction of the vector $\vec{v} = [\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$

(c) $f(x, y) = x^2 - y^2$ at $(x_0, y_0) = (-3, 3)$ in the direction of the vector $\vec{v} = [\frac{12}{13}, \frac{5}{13}]$

(d) $f(x, y) = \sin x \cos y$ at $(x_0, y_0) = (0, \pi)$ in the direction of the vector $\vec{v} = [-\frac{1}{2}, \frac{\sqrt{3}}{2}]$

5. Compute the four second-order partial derivatives of the following functions

(a) $f(x, y) = x^2y^3 - x \sin y$,

(g) $f(x, y) = xy^2z^3 - y \sin z$

(b) $f(x, y) = x\sqrt{y} - e^z \ln y$,

(h) $f(x, y) = x^y, x > 0$,

(c) $f(x, y) = (\sin x)^{\ln y}$,

(i) $f(x, y) = xy \ln(x^2 + y^2)$

(d) $u(x, y) = f(3xy, y^2x, x)$,

(j) $u(x, y, z) = f(z, xz^2, y)$

(e) $f(u, v) = e^{uv}$, gdzie $u = \ln x^2 + y^2, v = \arctan(\frac{y}{x})$

(f) $f(u, v) = \ln \frac{u}{v+1}$, gdzie $u = x \sin y, v = x \cos y$

6. Compute

(a) $\frac{\partial^5 f}{\partial x \partial y^4}$, for $f(x, y) = xe^{-y}$

(b) $\frac{\partial^5 f}{\partial z^2 \partial x \partial y}$, for $f(x, y, z) = \ln(x^2 + 2y - z)$

7. Find the Taylor polynomials about (x_0, y_0) for the following functions

(a) $f(x, y) = \sin^2(x + y), (x_0, y_0) = (\pi, \pi), n = 2$

(b) $f(x, y) = -x^2 + 2xy + 3y^2 - 6x - 2y - 4, (x_0, y_0) = (-2, 1), n = 3$

(c) $f(x, y) = \sin(x^2 + y^2), (x_0, y_0) = (0, 0), n = 3$

(d) $f(x, y) = \sin(x)e^{2y}, (x_0, y_0) = (0, 0), n = 3$

8. Find the Jacobian matrix (A -matrix) for the following functions

(a) $f(x, y) = (xy^2, 2x + y^2, 3xy)$

(b) $f(x, y) = x^4y^2$

(c) $f(x, y) = (xy^2, 2x + y^2, 3xy)$