Exercises and problems for Functions of Several Variables.

1. Determine and draw an area of the following functions

   (a) $f(x,y) = \dfrac{\sqrt{4-x^2-y^2}}{x^2+y^2-1}$

   (b) $f(x,y) = \ln(3-\sqrt{x+y})$

   (c) $f(x,y) = \sqrt{x\sin y}$

   (d) $f(x,y) = \arcsin(\sqrt{y-\sqrt{x}})$

   (e) $f(x,y) = \dfrac{3x}{2x-5y}$

   (f) $f(x,y,z) = \sqrt{x}+\sqrt{y-1}+\sqrt{z-2}$

   (g) $f(x,y,z) = \arcsin(x^2+y^2+z^2-2)$

2. Draw a contour diagram for the following functions and relate it to their graphs

   (a) $f(x,y) = 2-x^2-y^2$

   (b) $f(x,y) = \dfrac{1}{1+x^2+y^2}$

   (c) $f(x,y) = -\sqrt{9-y^2}$

   (d) $f(x,y) = \sqrt{x^2+y^2}$

   (e) $f(x,y) = \sin y$

3. Find the limits of the following functions as $(x,y) \longrightarrow (0,0)$.

   (a) $f(x,y) = e^{-x-y}$;

   (b) $g(x,y) = x^2+y^2$;

   (c) $h(x,y) = \frac{x}{x^2+1}$;

   (d) $i(x,y) = \frac{x+y}{2+\sin y}$;

   (e) $j(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$

   (f) $k(x,y) = \frac{x^2}{x^2+y^2}$

   (g) $l(x,y) = \frac{\sin xy}{x}$ as $(x,y) \longrightarrow (0,a)$

4. Are the following functions continuous at all points in the given regions?

   (a) $f(x,y) = \begin{cases} \sqrt{x^2+y^2}, & x \geq 0 \\ 2, & x < 0 \end{cases}$,

   (b) $f(x,y) = \begin{cases} \sin x, & x \in R, y \geq 0 \\ 1, & x \in R, y < 0 \end{cases}$.

   (c) $\frac{1}{x^2+y^2}$ on the square $-1 \leq x \leq 1$, $-1 \leq y \leq 1$;

   (d) $\frac{1}{x^2+y^2}$ on the square $1 \leq x \leq 2$, $1 \leq y \leq 2$;

   (e) $\frac{y}{x^2+2}$ on the disk $x^2+y^2 \leq 1$;

   (f) $\mathrm{tg}(xy)$ on the square $-2 \leq x \leq 2$, $-2 \leq y \leq 2$;

   (g) $\sqrt{2x-y}$ on the disk $x^2+y^2 \leq 4$.