

1. Compute

$$(a) \int_0^1 \left(\int_0^1 (x+y) dy \right) dx, \quad (b) \int_0^4 \left(\int_4^{12} x y dy \right) dx, \quad (c) \int_{-1}^1 \int_{x^2}^1 \int_0^2 (4+z) dz dy dx,$$
$$(d) \int_0^1 \int_0^{\sqrt{x}} \int_{1-x}^{2-2x} y dz dy dx,$$

2. Evaluate the integrals by reversing the order of integration

$$(a) \int_0^2 \left(\int_x^{2x} dy \right) dx, \quad (b) \int_1^2 \left(\int_{2-x}^{\sqrt{2x-x^2}} dy \right) dx, \quad (c) \int_0^1 \left(\int_{x^3}^{x^2} dy \right) dx, \quad (d) \int_1^e \left(\int_0^{\ln x} dy \right) dx,$$
$$(e) \int_{-2}^1 \left(\int_{y^2}^4 dx \right) dy, \quad (f) \int_0^1 \left(\int_x^{2-x^2} q dy \right) dx, \quad (g) \int_1^2 \left(\int_{\frac{1}{y}}^y z dx \right) dy.$$

3. Find integrals of the function over the region A:

- (a) $f(x, y) = x \cdot y$ and A is the rectangle bounded by curves $x = 0, x = a, y = 0, y = b$,
- (b) $f(x, y) = 2x + y - 1$ and A is the triangle with corners $A(1, 1), B(5, 3), C(5, 5)$,
- (c) $f(x, y) = \sin(x + y)$ and A is the region bounded by $y = 0, y = x, x + y = \frac{\pi}{2}$,
- (d) $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ and $A = \{ (x, y) \in R^2, a^2 < x^2 + y^2 \leq b^2, a > 0, b > 0 \}$,
- (e) $f(x, y) = x^2 + y^2 - a^2$ and $A = \{ (x, y) \in R^2, x^2 + y^2 \leq ax, a > 0 \}$,
- (f) $f(x, y) = y \sqrt{x^2 + y^2}$ and $A = \{ (x, y) \in R^2, x^2 + y^2 < 9, x < 0 \}$,
- (g) $f(x, y, z) = \frac{1}{1-x-y}$ and A is the region bounded by $x + y + z = 1, x = 0, y = 0, z = 0$,
- (h) $f(x, y, z) = (18x^2 + 8y^2)e^z$ and $A = \{ (x, y, z) \in R^3; \frac{x^2}{4} + \frac{y^2}{9} < 1, |z| < 2 \}$,
- (i) $f(x, y, z) = 2x + 3y - z$, and A is the rectangular box bounded by $x = 0, y = 0, z = 0, z = 3, x + y = 2$,
- (j) $f(x, y, z) = z \sin(x^2 + y^2)$ and A is the region bounded by $x = 0, y = 0, z = 0, z = 1, x^2 + y^2 = 1$,
- (k) $f(x, y, z) = ze^{-\frac{9x^2 + 4y^2}{2}}$ and $A = \{ (x, y, z) \in R^3, \frac{x^2}{4} + \frac{y^2}{9} \leq 1, 0 \leq z \leq 1 \}$,
- (l) $f(x, y, z) = xyz$ and A is the region bounded by $x^2 + y^2 + z^2 = 1, x = 0, y = 0, z = 0$.

4. Find the area bounded by the curves

$$(a) y^2 = x, x^2 = 8y, \quad (b) 3x^2 = 25y, 5y^2 = 9x.$$
$$(c) y = x^2 - 2x + 2, \text{ the tangent at } (3, 5), OY\text{-axes and } OX\text{-axes,}$$

5. Find the volume of the solid bounded by the following surfaces

- (a) $x^2 + y^2 + z^2 = x$,
- (b) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$,
- (c) $x = 0, x = 1, y = 2, y = 5, z = 2, z = 4$,
- (d) $x + y + z = 1, x = 0, y = 0, z = 0$,
- (e) $\frac{x^2}{4} + \frac{y^2}{9} < 1, |z| < 2$,
- (f) $x = 0, y = 0, z = 0, z = 1, x^2 + y^2 - 2y = 3$,
- (g) $x^2 + y^2 + z^2 = 1, x \geq 0, y \leq 0, z \in R$.