Polar,cylindrical and spherical coordinates

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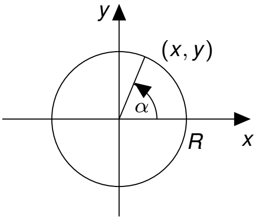
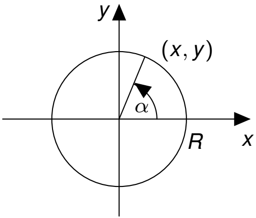
Polar coordinates



Let $A := \{(x, y); x^2 + y^2 \leq R\}$. Then, to compute $\int_A f \, dA$ we can use polar coordinates

$$\begin{cases} x = r\cos\alpha \\ y = r\sin\alpha \end{cases} \qquad r \in [0, R], \quad \alpha \in [0, 2\pi), \quad dA = r dr d\alpha,$$

Polar coordinates



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Thus,

$$\int_A f(x, y) \, dA = \int_0^{2\pi} \int_0^R f(x, y) \, r \, dr \, d\alpha.$$

Polar coordinates

In particular,

- If $A := \{(x, y) \in R^2;\ \ (x-a)^2 + (y-b)^2 = R^2\}$, then we use the following change of coordinates:

$$\begin{cases} x = a + r\cos\alpha \\ y = b + r\sin\alpha \end{cases} \qquad r \in [0, R], \quad \alpha \in [0, 2\pi), \quad dA = r\,dr\,d\alpha,$$

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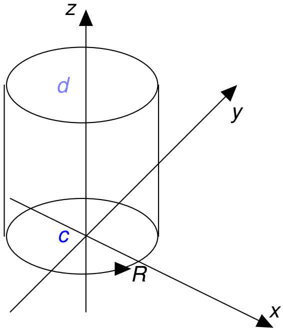
- If $A := \{(x,y) \in R^2;\ \ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\}$, then we use the following change of coordinates:

$$\begin{cases} x = a\,r\cos\alpha \\ y = b\,r\sin\alpha \end{cases} \qquad r \in [0,1], \quad \alpha \in [0,2\pi), \quad dA = abr\,dr\,d\alpha,$$

Cylindrycal coordinates

Let the set A will be a cylinder:



Cylindrycal coordinates

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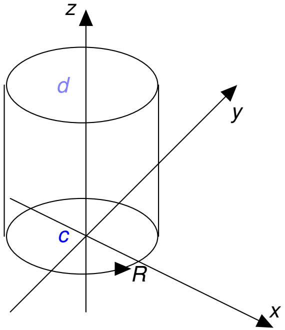

then, to compute $\int_A f \; dA$ we can use cylindycal coordinates

$$\begin{cases} x = r\cos\alpha \\ y = r\sin\alpha \\ z = z \end{cases} \qquad r \in [0, R], \quad \alpha \in [0, 2\pi), \quad z \in [c, d] \quad dA = r dr d\alpha dz,$$

Cylindrycal coordinates

In particular,

- If base of a cylider is a circle with a center at (a, b), then we use the following change of coordinates:

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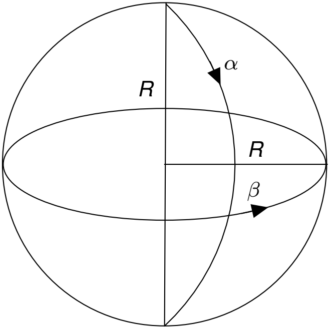
- If base of a cylider is an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then we use the following change of coordinates:

$$\begin{cases} x = a\, r\cos\alpha \\ y = b\, r\sin\alpha \\ z = z, \quad r \in [0, 1], \quad \alpha \in [0, 2\pi), \quad z \in [c, d] \quad dA = abrdrd\alpha dz, \end{cases}$$

Spherical coordinates



$$r \in [0, R]$$
$$\alpha \in [0, \pi]$$
$$\beta \in [0, 2\pi)$$

Let $A := \{(x, y); x^2 + y^2 + z^2 \leq R\}$. Then, to compute $\int_A f \, dA$ we can use spherical coordinates

$$\begin{cases} x = r \sin\alpha \cos\beta \\ y = r \sin\alpha \sin\beta \\ z = r \cos\alpha \end{cases} \qquad dA = r^2 \sin\alpha dr d\alpha d\beta,$$