

Testaw 2. Zad. 9e)

$$\lim_{x \rightarrow 4} \frac{\sin(4-x)}{\sin(5x-20)} \stackrel{[0/0]}{=} \lim_{x \rightarrow 4} \frac{\frac{\sin(4-x)}{4-x} \cdot (4-x)}{\frac{\sin(5x-20)}{5x-20} \cdot (5x-20)} =$$

$$(*) \lim_{x \rightarrow 4} \sin(4-x) = \sin(4-4) = \sin 0 = 0, \quad \lim_{x \rightarrow 4} \sin(5x-20) = \sin(5 \cdot 4 - 20) = 0$$

korzystamy ze wzoru: $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, a ścisłej, jego ogólniejszej postaci $\lim_{x \rightarrow x_0} \frac{\sin(g(x))}{g(x)} = 1$, jeśli $\lim_{x \rightarrow x_0} g(x) = 0$;

otynamyemy

$$(1) \lim_{x \rightarrow 4} \frac{\sin(4-x)}{4-x} = 1 \quad (\text{przyjmując } g(x) = 4-x, x_0 = 4)$$

oraz

$$(2) \lim_{x \rightarrow 4} \frac{\sin(5x-20)}{5x-20} = 1 \quad (\text{przyjmując } g(x) = 5x-20, x_0 = 4)$$

$$= \frac{\lim_{x \rightarrow 4} \frac{\sin(4-x)}{4-x}}{\lim_{x \rightarrow 4} \frac{\sin(5x-20)}{5x-20}} \cdot \lim_{x \rightarrow 4} \frac{4-x}{5x-20} \stackrel{(1)(2)}{=} \frac{1}{1} \cdot \lim_{x \rightarrow 4} \frac{4-x}{(-5)(4-x)} = -\frac{1}{5}$$

Testaw 2. Zad. 9f)

$$\lim_{x \rightarrow 0} \frac{\sin^2(3x) - \tan^2(3x)}{x^4} \stackrel{[0/0]}{=} \lim_{x \rightarrow 0} \frac{\sin^2(3x) - \frac{\sin^2(3x)}{\cos^2(3x)}}{x^4} =$$

$$(*) \lim_{x \rightarrow 0} (\sin^2(3x) - \tan^2(3x)) = \sin^2(3 \cdot 0) - \tan^2(3 \cdot 0) = 0; \quad \lim_{x \rightarrow 0} x^4 = 0; \quad \tan x = \frac{\sin x}{\cos x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin^2(3x)\cos^2(3x) - \sin^2(3x)}{\cos^2(3x)}}{x^4} = \lim_{x \rightarrow 0} \frac{\sin^2(3x)[\cos^2(3x) - 1]}{x^4 \cos^2(3x)} \stackrel{(**)}{=}$$

$$\left\{ \begin{array}{l} (*) \\ (*) \end{array} \sin^2(3x) + \cos^2(3x) = 1 \right\}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2(3x)[- \sin^2(3x)]}{x^4 \cos^2(3x)} = \lim_{x \rightarrow 0} \frac{\sin^4(3x)}{x^4} \cdot \frac{(-1)}{\cos^2(3x)} =$$

$$\left\{ \begin{array}{l} (1) \lim_{x \rightarrow 0} \frac{\sin^4(3x)}{(3x)^4} = \lim_{x \rightarrow 0} \left[\frac{\sin(3x)}{3x} \right]^4 = \left(\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \right)^4 \stackrel{(**)}{=} 1^4 = 1 \quad \left(\begin{array}{l} (*) \\ (*) \end{array} \right) \begin{array}{l} x_0 = 0 \\ g(x) = 3x \end{array} \end{array} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^4(3x)}{(3x)^4} \cdot \frac{(3x)^4}{x^4} \cdot \frac{(-1)}{\cos^2(3x)} = \lim_{x \rightarrow 0} \frac{\sin^4(3x)}{(3x)^4} \cdot \lim_{x \rightarrow 0} \frac{(3x)^4}{x^4} \cdot \lim_{x \rightarrow 0} \frac{(-1)}{\cos^2(3x)} =$$

$$\stackrel{(1)}{=} 1 \cdot \lim_{x \rightarrow 0} \frac{81x^4}{x^4} \cdot \frac{(-1)}{\cos^2(3 \cdot 0)} = 1 \cdot 81 \cdot \frac{(-1)}{1} = -81$$