The Banach Center Conference

Second Workshop on Nonlinearity and Geometry.

Darboux Days

Book of Abstracts

The Mathematical Research and Conference Center at Będlewo

Poland, April 13-19, 2008

Reciprocal transformations and Hamiltonian systems of hydrodynamic type

Simonetta Abenda, University of Bologna

Abstract

In this talk I consider hydrodynamic equations which admit an Hamiltonian structure of Dubrovin-Novikov type and study the effect of a reciprocal transformation (transformation of the independent variables) on the system. In particular I shall present some new results concerning necessary and/or sufficient conditions for which the reciprocal system is still of Dubrovin-Novikov type. I shall also present some applications to derive new families of flat metrics on Hurwitz spaces.

On isometric immersions of n-dimensional Labacthevsky space into (2n-1)-dimensional Euclidean space

Yuriy Aminov, Institute for Low Temperature Physics and Engineering, NAS, Kharkov

Abstract

The study of isometric immersions of n-dimensional Lobachevsky space L^n into Euclidean space E^{2n-1} from the local and global points of view is considered in the author's papers [1-10].

Let F^n be a regular submanifold in E^{2n-1} isometric to some simple connected domain of the Lobachevsky space L^n with a curvature equal to -1. Grassmann map $\psi : F^n \to G_{n-1,2n-1}$ correlates the (n-1)-dimensional space N passing through some fixed point $O \in E^{2n-1}$ with every point $x \in F^n$, the space N being parallel to normal space N_x of F^n at the point x (i.e. it corresponds to every point x some point of Grassmann manifold $G_{n-1,2n-1}$). The image of this map $\psi(F^n)$ we denote Γ^n. In terms of curvature coordinates the metric form of F^n can be written in the form

$$ds^2 = \sum_{i=1}^{n} \sin^2 \sigma_i du_i^2, \quad \sum_{i=1}^{n} \sin^2 \sigma_i = 1. \tag{1}$$

The Grassmann map ψ carries the coordinates u_i to Γ^n, and in these coordinates the metric of Γ^n is written as follows (see [2])

$$dl^2 = \sum_{i=1}^{n} \cos^2 \sigma_i du_i^2, \quad \sum_{i=1}^{2} \cos^2 \sigma_i = n - 1. \tag{2}$$

Hence it follows that Γ^n is a regular n-dimensional submanifold. The map for $n > 2$ increases the volume of any domain of F^n and the length of any asymptotic line. These properties imply the following result

Theorem 1. *If the Grassmann image Γ^n lies on a closed n-dimensional manifold and if the Grassmann map is finite-to-one, then the immersion of the full space L^n in E^{2n-1} has singularities.*

It is interesting to investigate different classes of immersions. One of such classes for $n = 3$ arises on condition that the Garssmann image is hyperplanar, i.e. $\Gamma^3 \subset E^9$.(In the general case $\Gamma^3 \subset E^{10}$.) The existence of local

isometric immersions with the hyperplanar Grassmann image is proved. In this case the author found the connection of the theory of isometric immersion with the theory of rigid body rotation with a fixed point in the central field of gravity and the Newton Law of gravity (see [3]). In some subcases the theorems about nonimmersion of full Lobachevsky space are proved. But the general case remains open.

The following question arises in a natural way: can the metric of the Grassmann image have a constant curvature ? The answer to this question for $n = 3$ is given in [6].

Theorem 2. *There is no local C^3 isometric immersion of L^3 into E^5 with constant curvature of the metric of the Grassmann image.*

In [10] we consider a $(n-1)$- parametric family of submanifolds F^n in E^{2n-1} with a constant negative curvature $K_0(F^n)$ in a ball D of the Euclidean space E^{2n-1}. We suppose that this family is included in some (2n-1)-orthogonal coordinate system $u_1, ..., u_{2n-1}$ as a family of coordinate submanifolds $u_{n+1} = const, ..., u_{2n-1} = const$. The author calls this system the Bianchi system of coordinates, if the first n coefficients H_i^2 of the metric form of the ambient space satisfy the following condition

$$\sum_{i=1}^{n} H_i^2 = 1. \tag{3}$$

Bianchi shows for $n = 2$ that the condition (3) satisfied automatically. The author has proved for $n = 3$ that for proving the next theorem it will suffice to demand the condition (3) only on two coordinate curves u_4, u_5 going through the center of the ball D. Besides, remark that on each submanifold F^n one can introduce the curvature coordinates, for which the condition (3) is true. In the paper [10] proved is

Theorem 3. *Assume that a ball of radius ρ in the Euclidean space E^{2n-1} carries a regular Bianchi system of coordinates such that $K_0(F^n) \leq -1$. Then*

$$\rho \leq \frac{\pi}{4}.$$

There exists an example of a regular Bianchi system in a ball $D \subset E^3$ with radius $\rho = \frac{1}{2}$.

As F^n is the submanifold with the flat normal connection, then on F^n there exists a field ξ of normal unit vectors parallel translated in the normal bundle. With the help of this field ξ we construct a map $\phi : F^n \to S^{2n-2}$ of the submanifold F^n into the unit sphere S^{2n-2}. We call the map ϕ spherical and denote its image $T(\xi)$. The metric of $T(\xi)$ has the following form:

$$(d\xi)^2 = \sum_{i=1}^{n} \cos^2 \sigma_i \cos^2 \phi_i du_i^2, \tag{4}$$

where ϕ_i is an angle between ξ and i-th principal vector of normal curvature k_i, $\quad i = 1, ..., n$. In the general case the spherical image cannot be regular and, moreover, it can degenerate in a submanifold of lower dimension than n.

We found a curvature tensor of the spherical image and proved a saddle character of spherical image, which considered as a submanifold in S^{2n-2}.

Theorem 4. *Under a spherical mapping the curvature lines are translated on the curvature lines of the spherical image, the asymptotic lines are translated on the asymptotic lines of submanifold $T(\xi) \subset S^{2n-2}$. The length of asymptotic lines is preserved under this mapping.*

References

1. Aminov Y.A. On the immersion of domains of n-dimensional Lobachevskii space in (2n-1)-dimensional Euclidean space. Soviet Mat. Dokl.v.18 (1977), N 5, p.12-1213.

2. Aminov Y.A. Isometric immersions of domains of n-dimensional Lobachevsky space in (2n-1)-dimensional Euclidean space. Matem. Sbornik, v.111 (1980) N 3, Math.USSR Sbornik v.39 (1981), N 3.

3. Aminov Y.A. Isometric immersions of domains of three-dimensional Lobachevsky space in five-dimensional Euclidean space and the motion of a rigid body. Matematichesky Sbornik , v.122(1983), vyp. 1, Math.USSR Sbornik v.50(1985), N 1, p.11-30.

4. Aminov Y.A. The property of the Grassmann image of local immersion of three-dimensional Lobachevsky space into five-dimensional Euclidean space. (Russian)Ukrainian geometrical Sbornik,vyp. 27 (1984), p.3-11, Sbornik Mathem. 197:2, p.139-152.

5. Aminov Y.A. Isometric immersions with flat normal connection of domains of n-dimensional Lobachevsky space into Euclidean spaces. A model of a gauge field.Matematicheski Sbornik, t. 137 (1988), vyp. 3. Math.USSR Sbornik , v.65 (1990) N 2, p.279-303.

6. Aminov Y.A. Geometry of the Grassmann image of a local isometric immersions of Lobachevsky n-dimensional space in (2n-1)-dimensional Euclidean space, Matematicheskii Sbornik 188:1, (1997), p.3-8.

7. Aminov Y.A. New Ideas in Differential Geometry of submanifolds, Kharkiv,Acta, 2000.

8. Aminov Y.A. The Geometry of Submanifolds , Amsterdam, Gordon and Breach Academic Publishing House, 2001.

9. Aminov Y.A. The expression of volume of asymptotic parallelepiped (Russian), Math. fizika, analiz, geometria, (2002) v. 9 N 4, p.519-532.

10. Aminov Y.A. Families of submanifolds of constant negative curvature of many-dimensional Euclidean space. Matematicheski Sbornik 197:2, (2006), 3-16.

Backlund transformations and solutions for integrable lattice equations

James Atkinson, University of Leeds

Abstract

New Backlund transformations for some known multidimensionally consistent lattice equations will be discussed. The equations considered are mostly those contained in the Adler-Bobenko-Suris classification. It will be described how, through new auto-Backlund transformations, particular pairs of equations admit a kind of duality in which each equation emerges as the permutability condition for Backlund transformations that relate solutions of the other. It is also described how some pairs of apparently distinct equations admit a Backlund transformation that connects them. The application of these Backlund transformations to the construction of new solutions for the lattice equations is also discussed.

Physical applications of twistors

Iwo Białynicki-Birula, Center for Theoretical Physics PAS, Warsaw

Abstract

A formulation of relativistic wave mechanics is proposed in terms of twistors. This formulation has practical applications because it enables one to generate the solutions of the Maxwell, Weyl, and other wave equations for massless particles. The key role in this formulation is played by a $4+4$ dimensional phase-space. Twistors arise as a convenient parametrization of this phase-space.

From Lie-Poisson structures to integrable systems. Classical R-matrix and deformation quantization

Maciej Błaszak, Adam Mickiewicz University, Poznań

Abstract

It is a review lecture which presents how to construct inegrable ODE's and PDE's using the R-matrix formalism on Lie algebras. In the first part of the lecure a general information about the formalism are presented, including the construction of Lax hierarchy and its R-Lie-Poisson structure. In the second part of the lecture a particular class of examples is presented. Actually, first is considered an example of Poisson algebras leading to integrable dispersionless field systems. Then, various deformations of Poisson algebras are constructed leading to dispersive integrable continuous and discrete field systems.

The Darboux transformation of conformal immersions and its discrete analogue

Christoph Bohle, Technical Universitaet Berlin

Abstract

I explain how the classical Darboux transformation of isothermic surface can be generalized to arbitrary conformal immersions into 4-space. The resulting evolution is a time-discrete version of the Davey-Stewartson flow on the space of confomal immersions. An analogous notion of Darboux transformation for discrete surfaces can be defined. It turns out to be a combinatorial reinterpretation of the space and time-discrete Davey-Stewartson flow studied by Konopelchenko and Schief. In case of immersed (smooth or discrete) tori, the space of Darboux transforms can be desingularized to a Riemann surface, the spectral curve of the immersion.

Isochronous systems are not rare

Francesco Calogero, University of Rome „La Sapienza"

Abstract

A (classical) dynamical system is called isochronous if it features an open (hence fully dimensional) region in its phase space in which all its solutions are completely periodic (i. e., periodic in all degrees of freedom) with the same fixed period (independent of the initial data, provided they are inside the isochrony region). When the isochrony region coincides with the entire phase-space one talks of entirely isochronous systems. A trick is presented associating to a dynamical system a modified system depending on a parameter so that when this parameter vanishes the original system is reproduced while when this parameter is positive the modified system is isochronous. This technique is applicable to large classes of dynamical systems, justifying the title of this talk. An analogous technique, even more widely applicable - for instance, to any translation-invariant (classical) many-body problem - transforms a real autonomous Hamiltonian system into an entirely isochronous real autonomous Hamiltonian system. The modified system is of course no more translation-invariant, but in its centre-of-mass frame it generally behaves quite similarly to the original system over times much shorter than the isochrony period T (which may be chosen at will). Hence, when this technique is applied to a "realistic" many-body Hamiltonian yielding, in its centre of mass frame, chaotic motions with a natural time-scale much smaller than (the chosen) T, the corresponding modified Hamiltonian shall yield a chaotic behavior (implying statistical mechanics, thermodynamics with its second principle, etc.) for quite some time before the entirely isochronous character of the motion takes over hence the system returns to its initial state, to repeat the cycle over and over again. We moreover show that the quantized versions of these modified Hamiltonians feature infinitely degenerate equispaced spectra. Analogous techniques are applicable to nonlinear evolution PDEs, but in this talk there will be no time to cover this aspect. The material presented is a synthesis of work done over the last 10 years with several collaborators, as reviewed in the 264-page monograph entitled Isochronous systems, just published by Oxford University Press.

Differential Calculus for Fedosov Deformation Quantization

Michał Dobrski, Technical University of Lódź

Abstract

Using Darboux coordinates and the theorem on local isomorphism of any deformation quantization to the trivial (Moyal) *-algebra we propose a differential calculus for arbitrary Fedosov's *-algebra. Some comments on Yang-Mills fields in noncommutative spaces with a point-dependent deformation parameter are also given.

On theory of three-orthogonal surfaces

Valery Driuma, Institute of Mathematics and Informatics, AS Moldova, Kishinev

Abstract

The properties of three-orthogonal surfaces defined by solutions of KdV-equation are discussed.

The solutions of Cayley-Darboux equation are considered.

Six-dimensional extensions of 3-dim Riemannian metrics defined by the KdV-equation and applications in theory of Einstein equations are studied.

Metricity in projective geometry

Maciej Dunajski, University of Cambridge

Abstract

Cover a plane with curves, one curve through each point in each direction. How can you tell whether these curves are the geodesics of some metric?

This problem gives rise to a certain closed system of partial differential equations and hence to obstructions to finding such a metric. It has been an open problem for at least 80 years. Surprisingly it is harder in two dimensions than in higher dimensions. I shall present a solution obtained jointly with Robert Bryant and Mike Eastwood.

Twistor theory and differential equations

Maciej Dunajski, University of Cambridge

Abstract

Twistor Theory was originally proposed by Roger Penrose as a way to unify quantum mechanics and general relativity. Its status as a physical theory remains unclear but it found unexpected applications in mathematics - from differential geometry and representation theory to integrable systems.

I will give an elementary introduction to the subject concentrating on applications to differential equations.

Second order quasilinear PDEs and conformal structures in projective space (joint work with P.A. Burovskiy and S.P. Tsarev)

Eugene Ferapontov, Loughborough University

Abstract

We investigate second order quasilinear equations of the form

$$f_{ij}u_{x^ix^j} = 0$$

where u is a function of n independent variables $x^1, ..., x^n$, and the coefficients f_{ij} are functions of the first order derivatives $p_1 = u_{x^1}, ..., p_n = u_{x^n}$. We show that the natural equivalence group of the problem is isomorphic to $SL(n+1, R)$ which acts by projective transformations on the space P^n with coordinates $p_1, ..., p_n$. The coefficient matrix f_{ij} defines on P^n a conformal structure $f_{ij}(\boldsymbol{p})dp_idp_j$. In this paper we concentrate on the case $n = 3$, although some results hold in any dimension. The necessary and sufficient conditions for the integrability of such equations by the method of hydrodynamic reductions are derived. These conditions constitute a complicated over-determined system of PDEs for the coefficients f_{ij}, which is in involution. We prove that the moduli space of integrable equations is 20-dimensional. Based on these results, we show that any equation satisfying the integrability conditions is necessarily conservative, and possesses a dispersionless Lax pair. Reformulated in differential-geometric terms, the integrability conditions imply that the conformal structure is conformally flat, and possesses an infinity of 3-conjugate coordinate systems of null curves. Integrable equations provide an abundance of explicit examples of such conformal structures expressible in terms of elementary functions, elliptic functions and modular forms.

Françoise

Gaston Darboux and his influence on Differential Geometry and Dynamical Systems

Jean-Pierre Françoise, Université P.-M. Curie, Paris

Abstract

This talk will be devoted to present the influence of Gaston Darboux on the development of Differential Geometry in France and in Europe via his scientific contributions and his students. The Mathematics Genealogy project shows he had two french students Emile Borel and Elie Cartan and one polish student Stanislaw Zaremba. They cummulate 4995 descendants to these days...After some review of his most representative contributions, we will focus on his relationship with Integrability.

GL(2,R) geometry of ODEs

Michał Godliński, Institute of Mathematics PAS, Warsaw

Abstract

This talk is based on joint work with Pawel Nurowski, `arXiv:0710.0297`.

We show that on the solution space of every 5th order ODE satisfying certain three nonlinear differential conditions there exists (locally) a five dimensional $\mathbf{GL}(2,\mathbb{R})$ geometry, i.e. a geometry associated with the 5-dimensional irreducible representation of $\mathbf{GL}(2,\mathbb{R})$. This is a special Weyl geometry in signature $(3,2)$ having the structure group reduced from $\mathbf{CO}(3,2)$ to $\mathbf{GL}(2,\mathbb{R})$. The reduction is obtained by means of a conformal class of totally symmetric 3-tensors. The conditions for the existence of the structure are lower order equivalents of the Doubrov-Wilczynski conditions found recently by Boris Doubrov.

Among all $\mathbf{GL}(2,\mathbb{R})$ geometries we distinguish a subclass which we term 'nearly integrable $\mathbf{GL}(2,\mathbb{R})$ geometries'. These define a unique $\mathfrak{gl}(2,\mathbb{R})$ connection which has totally skew symmetric torsion. This torsion splits onto the $\mathbf{GL}(2,\mathbb{R})$ irreducible components having respective dimensions 3 and 7.

The $\mathbf{GL}(2,\mathbb{R})$ geometry on the solution space of every 5th order ODE satisfying the three conditions is very special: it is nearly integrable, and the skew symmetric torsion of its unique $\mathfrak{gl}(2,\mathbb{R})$ connection belongs to the 3-dimensional irreducible representation of $\mathbf{GL}(2,\mathbb{R})$. We provide nontrivial examples of 5th order ODEs which possess this $\mathbf{GL}(2,\mathbb{R})$ geometry.

We also outline the theory and the basic properties of $\mathbf{GL}(2,\mathbb{R})$ geometries associated with n-dimensional irreducible representations of $\mathbf{GL}(2,\mathbb{R})$ in $6 \leq n \leq 9$. In particular we give conditions for an nth order ODE to possess this geometry on its solution space.

Goldstein

Renormalizaton of the Laurent expansion as a tool in the Hirota bilinear method

Piotr Goldstein, Andrzej Soltan Institute for Nuclear Studies, Warsaw

Abstract

The first stage of the Hirota bilinear method (which has become a basic tool for construction of soliton solutions to partial differential equations) is a rational substitution leading to the bilinear variables. It is generally believed that finding the proper substitution is an art rather than an algorithm. An application of the singularity analysis to make the approach more systematic was suggested in the classical work [1] and generalized in [2]. The approach was based on the fact that movable singularities of the solution were zeros of the denominator(s) in the rational substitution. If the equation had at least the partial Painlevé property, its solution could be expanded into a Laurent series about a movable singularity manifold like in the Painlevé test. Then the principal part of the expansion might be used for the substitution.

Our approach is an extension of the method of [2]. Namely, it often happens that the principal part does not have the proper bilinear form. Here we suggest how the Laurent series can be "renormalized" by a singularity-preserving transformation of the expansion variable. The renormalization results in an inclusion of the higher order terms to the principal part of the expansion. The new principal part may be expressed in terms of the Hirota bilinear differential operator or its multilinear generalization.

[1] J. Weiss, M. Tabor and G. Carnevale, *J. Math. Phys.* **24** (1983) 522.
[2] P.G. Estevez, P.R. Gordos, L. Martinez Alonso and E. Medina Reus, *J. Phys A: Math. Gen.* **26**, 1915 (1993).

On the quasiclassical geometry

Boris Konopelchenko, University of Salento, Lecce

Abstract

What it could be? The formulation of this question is motivated by old studies of physical phenomena for corrugated surfaces with slow modulation and by recent results on quasiclassical limit of the gauge/string correspondence as well as by the well-known interrelations between the various types of geometry from the classical (affine, projective etc.) to the tropical one.

In attempt to provide very preliminary and partial answer we discuss quasiclassical limit for the five known geometrical problems. They are:

1. Quasiclassical motion of space curves governed by the quasiclassical Da Rios system or the quasiclassical NLS equation which are equivalent to the well-known hydrodynamical Benney system.

2. Quasiclassical version of the celebrated Darboux system describing conjugate nets.

3. Quasiclassical limit of the Menelaus lattice governed by an equation which coincides with the generating equation for the dKP hierarchy.

4. Quasiclassical generalized Weierstrass representations which describe highly oscilatting surfaces in R^3 and R^4 with slow modulation.

5. Quasiclassical limit of soliton surfaces given by A. Sym formulae.

One observes thath all these examples exhibit certain common properties characterized by a small value of ratio of two characteristic scales for the problem. These common properties may constitute (at least some) basic ingredients of the quasiclassical geometry.

On the geometry of plane distributions on 3-manifolds

Volodymyr Kruglov, Institute for Low Temperature Physics and Engineering, NAS, Kharkov

Abstract

We study different notions of curvature of the plane distributions on a 3-manifold. We show, that under assumption of non-integrability it is easy to manipulate the curvature. We prove that if the distribution is a contact structure, then every negative function is realized as a sectional curvature of that distribution. As a corollary we answer the question of John Etnyre about the uniformization of contact structures on 3-manifolds.

Darboux transformation on conformal maps

Katrin Leschke, University of Leicester

Abstract

The classical Darboux transformation can be extended to a transformation on conformal maps from a Riemann surface into the 4-sphere by relaxing the enveloping condition. I will discuss that this new transformation preserves special surface classes such as CMC surfaces and Hamiltonian Stationary Lagrangian surfaces. In the case of tori, the set of closed Darboux transforms can be made into a Riemann surface, which is in the special case of CMC tori the spectral curve as defined by Hitchin. In this case, the classical Darboux transforms then correspond to special points on the spectral curve.

Miura Transformations and Symmetries of nonlinear equations defined on a lattice

Decio Levi, Universitŕ Roma Tre

Abstract

After reviewing the construction of Lax pairs for the integrable lattice equations belonging to the Adler-Bobenko-Suris (ABS) list, we provide explicit Miura transformations mapping their scalar spectral problems into the discrete Schr¨odinger spectral problem associated with Volterra-type equations. We show that the ABS equations correspond to Bäcklund transformations for some particular cases of the discrete Krichever-Novikov equation found by Yamilov. This enables us to construct new generalized symmetries for the ABS equations.

Stäckel systems, hydrodynamic systems and new solutions of coupled KdV hierarchy

Krzysztof Marciniak, Linköping University

Abstract

We consider Stäckel separable systems of Benenti type and a class of weakly-nonlinear hydrodynamic systems constructed with the help of Killing tensors related with our Stäckel systems (we call them Killing systems of Benenti type). We show that both families of systems share the same class of multi-time solutions. Within this class of solutions our Killing systems turn into soliton hierarchies: either coupled Korteveg-de Vries (cKdV) or coupled Harry Dym hierarchy. The mentioned solutions turn then into wide classes of solutions of these hierarchies. We illustrate these ideas in the case of cKdV hierarchy producing its new finite-gap, rational and implicit solutions.

Marvan

On recursion operators of the IGSG equation

Michal Marvan, Silesian University in Opava

Abstract

We shall discuss Guthrie type recursion operators of integrable systems in general and recursion operator of the intrinsic generalized sine-Gordon equation in particular.

Some examples integrable geometrical flows in 2+1 dimensions

Ratbay Myrzakulov, Eurasian National University, Astana

Abstract

Geometrical flows (GF) play an important role in modern mathematics and physics. In this letter we have considered some integrable isotropic GF – Ricci flows (RF) and mean curvature flows (MCF) – which are related with integrable Heisenberg ferromagnets. In 2+1 dimensions, these GF have a singularity at $t = t_0$. This talk is based on a joint work with N.S.Serikbaev, Zh.M.Bitibaeva, K.K.Yerzhanov (ArXiv: 0804.0837).

On the surface in 5-dimensional Euclidean space which belong to the hypersphere

Yuryi Aminov and Iana Nasiedkina
Institute for Low Temperature Physics and Engineering, NAS, Kharkov

Abstract

Conditions of accessory of a 2-dimensional surface F^2 in space E^5 to 4-dimensional sphere S^4 are obtained for a hyperbolic case. It is known, that for F^2 indicatrix of normal curvature for every point $x \in F^2$ in n-dimensional space E^n, is a flat curve. More precisely it is an ellipse. The plane of this ellipse we denote α_x

Theorem. . *If the surface $F^2 \in E^5$ belongs to a sphere S^4 with the radius R, then for each point $x \in F^2$ the plane α_x lies on the constant distance $\frac{1}{R}$ from this point x.*

The inverse theorem we prove under additional conditions. In the further we'll distinguish three cases:

1) the elliptic case: the projection $\bar{x}$ of the point x on the plane α_x lies inside of this ellipse,

2) the parabolic case: the projection $\bar{x}$ lies on the ellipse,

3) the hyperbolic case: the projection $\bar{x}$ lies outside of the ellipse .

We consider the surface with the hyperbolic case.

Let's define on a surface by the invariant-geometrical way some net of curves. In this case the point $\bar{x}$ lies outside of the ellipse. There exist two tangents straight lines to the ellipse through the point $\bar{x}$. Points of a contact with the ellipse we denote P and Q. Let k_{nP} and k_{nQ} be the vectors of normal curvature of a surface F^2 corresponding to each of these points. Some direction in a tangent plane to the surface corresponds to a vector k_{nP}, such that the vector of normal curvature for this direction coincides with k_{nP}. We consider a field of these directions in a neighborhood of the point x. This field generates a family of integrated curves. Similarly, we obtain other family of curves using field of vectors of normal curvature k_{nQ}. On the surface F^2 these two families form a net of curves which we name specially hyperbolic one.

Theorem. . *Let the surface $F^2 \subset E^5$ with non degenerate ellipse of normal curvature belongs to a hyperbolic class. Let D is triangular domain on F^2, which is limited by two curves η_1 and η_2 from the different families of the specially hyperbolic net, which are going from one point, and by some curve τ,which crosses η_1 and η_2. Let for each point $x \in D$ the plane α_x is situated on constant distance γ from a point x, .*

Let the following boundary conditions on τ are satisfied:

ŕ) the straight-lines orthogonal to the plane α_x through $x \in \tau$, for all points $x \in \tau$ intersects in a one point. (from a) condition it is followed, that τ is the spherical curve)

b)for the field of unite normals n_3 orthogonal to the plane α_x and any normal n will be satisfied $(n_{3s}, n)|_\tau = 0$, where $\frac{\partial}{\partial s}$ is differentiation in a direction orthogonal to τ.

Then the domain D of the surface F^2 belongs to some sphere S^4 with radius $\frac{1}{\gamma}$.

Nijhoff

On Soliton Solutions of Integrable Lattice equations

Frank Nijhoff, University of Leeds

Abstract

In recent years integrable lattice equations, i.e. partial difference equations analogues of soliton PDEs, have gained a lot of interest. Whereas lattice equations of KdV type, and their soliton solutions have been known since the early 1980s, the more recent classification result of *scalar* quadrilateral lattices by Adler, Bobenko and Suris, which are integrable in the sense of "multidimensionally consistency", has provided a number of novel examples. In this talk an overview will be given of some of the old as well as novel results for soliton solutions for these equations.

Darboux transformations for noncommutative systems

Jonathan Nimmo, University of Glasgow

Abstract

This talk will review some recent applications [2, 3, 4] of quasideterminants [1] to the use of Darboux transformations to construct exact solutions to noncommutative versions of a number of classical integrable systems.

[1] Gelfand, I. M. and Retakh, V. S. Funktsional. Anal. i Prilozhen., 25, 13 (1991).
[2] Gilson, C. R. and Nimmo, J. J. C. J. Phys. A, 40, 3839 (2007).
[3] Gilson, C. R., and Sooman, C. M. J. Phys. A, 41 (2008).
[4] Li, C. X. and Nimmo, J. J. C. Proc. R. Soc. Lond. 646, 951 (2008).

Panasyuk

Inner symmetries of bihamiltonian structures and Lie pencils

Andriy Panasyuk, University of Warsaw

Abstract

The starting point will be an n-dimensional free rigid body, whose inertia matrix has nonsimple spectrum, i.e. possessing continuous symmetries (eg. ball or "cucumber"). To integrate such a system one needs to enlarge the family of first integrals obtained by the standard methods (for instance the family of Manakov integrals). One needs to add the noetherian integrals related to the symmetries.

This picture will be considered in the more general context of bihamiltonian structures with "inner" symmetries. A criterion of completeness of the corresponding enlarged family of functions will be presented. This result will be applied to integrable systems related to the so-called Lie pencils, i.e. linear families of Lie brackets on a vector space.

Integrability of the Egorov hydrodynamic type systems

Maxim Pavlov, P.N. Lebedev Physical Institute RAS, Moscow

Abstract

We consider three distinct classes of PDE's: systems described Egorov (potential) metrics (known as Binachi–Darboux–Lame–Egorov); Egorov hydrodynamic type systems, associativity equations (known as WDVV equations in topological field theories). Any solution of the WDVV equation determines N commuting Egorov hydrodynamic type systems determined up to N^2 arbitrary constants; these hydrodynamic type systems determine corresponding orthogonal curvilinear coordinate net.

Two mode solutions to hydrodynamic type equations

Zbigniew Peradzyński, University of Warsaw

Abstract

The story begins with Euler equations of compressible fluid. They form a first order homogenous quasi-linear system of equations for four unknown functions. Special solutions, e.g. simple waves were known since Riemann. Simple waves have one –dimensional hodograph. It is, the rank of the Jakcobi matrix of the solution is equal to 1 (these are one mode solutions). A number of authors e.g. Giese, Yanienko, Burnat..., have studied solutions with two dimensional hodograph (two mode solutions).J.H. Giese has noticed that there are two types of solutions hyperbolic double waves and elliptic double waves. M. Burnat has found the necessary and sufficient conditions in terms of "wave vectors" for the existence of hyperbolic double waves. Yanienko has derived an equation characterizing the two dimensional hodograph in both (hyperbolic and elliptic) cases. We study the problem of existence of two mode solutions for a general quasi-linear homogenous system. Two mode solutions can be sought of as representing the interaction of two Riemann waves without generating any new wave [1]. We ask the question: under what conditions this type of interaction is generic. Then using geometrical approach we derived the generalization of Yanienko equation in geometrical terms.

[1] Z. Peradzynski "Geometry of Riemann waves". [In] L. Debnath ed., Advances in Nonlinear Waves, Pitman (1985), (244–285).

LYZ equation, Hitchin system and Painleve III

Prim Plansangkate, University of Cambridge

Abstract

We give a gauge invariant characterisation of the symmetry reduction from the anti-self-dual Yang-Mills system on R^4 with gauge group $SU(2,1)$ to the Loftin-Yau-Zaslow (LYZ) equation

$$\psi_{z\bar{z}} + 1/2e^{\psi} + |U|^2 e^{-2\psi} = 0, \quad U_{\bar{z}} = 0,$$

which arises in the context of Strominger-Yau-Zaslow conjecture in Mirror Symmetry. The radially symmetric solutions of the LYZ equation are characterised by solutions of Painleve III with special values of parameters.

On the differential-geometric de Rham-Hodge aspects of the integrabilty

Anatoliy Prykarpatsky, National Academy of Sciences, Lviv

Abstract

The differential-geometric properties of generalized de Rham-Hodge complexes naturally related with integrable multi-dimensional differential systems of M. Gromov type are analyzed. The geometric structure of Chern type characteristic classes are studied, special differential invariants of the Chern type are constructed. The integrability of multi-dimensional nonlinear differential systems on Riemannian manifolds is discussed.

Weak hyperheavenly spaces and Walker spaces

Maciej Przanowski, Technical University of Lódź

Abstract

We show a close relation between hyperheavenly spaces introduced by Plebanski and Robinson in the context of complex relativity and Walker spaces in four dimensions. In particular some results of Walker geometry are presented within the 2-spinor language used by Boyer,Finley,Plebanski and collaborators in the hyperheavenly space theory.

Hamiltonian structure for reductions of the Benney system

Andrea Raimondo, Imperial College, London

Abstract

We show how to construct the Hamiltonian structures of any reduction of the Benney chain (dKP) starting from the family of conformal maps associated to it.

From Da Rios' equations to integrable vortex dynamics

Renzo Ricca, University of Milano-Bicocca

Abstract

In this paper we rapidly review 100 years of work on Da Rios equations and present new results on geometric aspects of integrable vortex dynamics. Much of the progress on integrable curve dynamics is rooted in the original work of Da Rios (1906) on three-dimensional motion of thin vortex filaments by the so-called localized induction approximation (LIA for short) and intrinsic dynamics, in terms of curvature and torsion of the filament in space. This work remained unnoticed for almost 100 years, and for nearly 60 years no other progress was made. But after Arms & Hama's re-discovery of LIA (1965) and Betchov's rediscovery (1965) of Da Rios' original intrinsic equations, an amazing new progress took place. By using Madelung transformation, Hasimoto (1972) showed that Da Rios' intrinsic equations are coupled into the Non-Linear Schrödinger (NLS) equation. Ricca, who contributed to the re-discovery (1991, 1996) of Da Rios' original work, extended the original intrinsic equations to their full generality, and by using results of Zakharov & Shabat (1972) gave soliton invariants an interpretation in terms of global geometric quantities. A recursive operator theory, developed by Langer & Perline (1991) classified the LIA dynamical hierarchy in terms of a wide family of integrable equations and conditions for integrable curve dynamics were found by Nakayama, Segur & Wadati (1992). Meanwhile solutions in terms of solitary waves in the shape of torus knots were found by Kida (1981), Keener (1990) and Ricca (1993), the latter being then tested and used in superfluid calculations (1999), as well as new quasi-static solutions to Da Rios' equations by Lipniacki (2002). In this rich scenario, we present new results that relate areas of projected graphs and integrable vortex dynamics using structural complexity analysis.

[1] Ricca, R.L. (1991) Rediscovery of Da Rios equations. Nature 352, 561-562.
[2] Ricca, R.L. (1996) The contributions of Da Rios and Levi-Civita to asymptotic potential theory and vortex filament dynamics. Fluid Dyn. Res. 18, 245-268.
[3] Ricca, R.L. (Ed.) 2001 An Introduction to the Geometry and Topology of Fluid Flows. NATO ASI Series II, 47. Kluwer.

Santini

Recent analytic results on integrable multidimensional PDEs connected with one - parameter families of commuting vector fields

Paolo Maria Santini, University of Rome „La Sapienza"
(joint work with S. V. Manakov)

Abstract

Well-known integrable multidimensional PDEs, like the second heavenly equation of Plebanski and the dispersionless Kadomtsev - Petviashvili (dKP) equation (describing respectively self-dual Einstein fields and two - dimensional shallow water waves near the shore), arise as the commutation of one - parameter families of vector fields. We develop the associated Inverse Spectral Transform and use it i) to solve the Cauchy problem for the above models; ii) to study the longtime behavior of their solutions; iii) to characterize classes of implicit solutions and iv) give a complete analytic description of the gradient catastrophe, at finite time and in the longtime regime, of localized two dimensional water waves evolving according to the dKP equation.

Aspects of integrable discrete differential geometry

Wolfgang Schief, Technical Universitaet Berlin

Abstract

Discretisation procedures which preserve integrable structure in objects of classical differential geometry rely on the interplay between geometric and algebraic methods. In this introductory talk, we focus on surfaces which are naturally parametrised in terms of curvature coordinates and embark on a discussion of how far the classical theory may be translated into the discrete language.

Generalized Stäckel Transform and Integrability

Artur Sergyeyev, Silesian University in Opava

Abstract

We present a multiparameter generalization of the Stäckel transform, also known as the coupling-constant metamorphosis, and show that under certain conditions this generalized Stäckel transform preserves the Liouville integrability, noncommutative integrability and superintegrability. The corresponding transformation for the equations of motion proves to be nothing but a reciprocal transformation of a special form, and we investigate the properties of this reciprocal transformation. This is joint work with Maciej Blaszak. For more details, see arXiv:0706.1473.

Silindir Yantir

R-matrix approach to integrable systems on time scales

Burcu Silindir Yantir, Adam Mickiewicz University, Poznań

Abstract

The general unifying framework for integrable soliton-like systems on time scales is introduced. The R-matrix formalism is applied to the algebra of δ-differential operators through which one can construct infinite hierarchy of commuting vector fields. The theory is illustrated by two infinite-field integrable hierarchies on time scales which are difference counterparts of KP and mKP hierarchies. The difference counterparts of AKNS and Kaup-Broer soliton systems are constructed as related finite-field restrictions.

A general integrable model for elastic rods

Matteo Sommacal, University of Perugia

Abstract

The study of elastic deformations in thin rods has recently seen renewed interest due to the close connection between these systems and coarse-grained models of widespread application in life- and material- sciences. Until now, the analysis has been restricted to the solution of equilibrium equations for continuous models characterized by constant Young and shear moduli and/or by isotropic rod section. We will show that integrable, indeed solvable, equations are obtained under more general conditions and that regular solutions emerge from reasonable choices of elastic stiffnesses. This work has been carried out in collaboration with Mario Argeri and Vincenzo Barone (Universitŕ "Federico II" di Napoli) and Silvana De Lillo and Gaia Lupo (Universitŕ degli Studi di Perugia).

Darboux's greatest love

Antoni Sym, University of Warsaw

Abstract

The lecture is centered around the early papers of J.G.Darboux on orthogonal coordinates.These papers are important since they contain germs of the subsequent work.It is also an "excursion" to the origins of the algebraic geometry (circular points at infinity,singularities, generalized foci in the sense of Plucker,etc.).

Central extensions of universal hierarchy: (2+1)-dimensional bi-Hamiltonian systems

Błazej Szablikowski, Adam Mickiewicz University, Poznań

Abstract

We consider the R-matrix formalism on the semi-direct product of the loop algebra of vector fields on circle and its dual. On these 'double' algebra one can introduce natural ad-invariant symmetric bilinear product. Through central extensions, given by appropriate two-cocycles, additional independent variable as well as dispersion are introduced. As result (2+1)-dimensional integrable hierarchies together with their (explicit) bi-Hamiltonian structures (without operand formalism) are constructed. Examples will be provided.

L-isothermic and L-minimal surfaces

Adam Szereszewski, University of Warsaw

Abstract

L-isothermic surfaces can be associated to solutions of the second order linear nonhomogeneous differential equation. This approach allows to construct the Weierstrass representation of L-minimal surfaces. Bäcklund transformation of L-isothermic surfaces will be also presented.

Periodic Box-Ball System: an Integrable Cellular Automaton

Tetsuji Tokihiro, The University of Tokyo

Abstract

The Box-Ball System (BBS) is a discrete dynamical system of balls moving in a one dimensional array of boxes, which is a generalization of the soliton cellular automaton proposed by Takahashi and Satsuma in 1990. The BBS is obtained from an integrable partial differential (or difference) quation through a limiting procedure called ultradiscretization. It has sufficient number of conserved quantities as an integrable dynamical system and its initial value problem can be solved just as soliton equations. The BBS has also important relation to integrable lattice models, in particular those with symmetry of quantum algebras. In this talk, we mainly consider the BBS with periodic boundary condition, and show its important mathematical aspects such as the relation to the Riemann hypothesis, the string hypothesis in Bethe Ansatzs method, and the Jacobian of hyperelliptic curves.

A geometric approach to Tropical Weyl group actions and q-Painleve equations

Teruhisa Tsuda, Kyushu University, Fukuoka

Abstract

Starting from certain rational varieties blown-up from $(\mathbb{P}^1)^N$, we construct a tropical (or subtraction-free birational) representation of Weyl groups as a group of pseudo-isomorphisms of the varieties. We develop an algebro-geometric framework of tau-functions as defining polynomials of exceptional divisors on the varieties. In the case where the corresponding root system is of affine type, our construction yields (higher order) q-difference Painleve equations. This talk is based on a joint work with Tomoyuki Takenawa (`arXiv:math.AG/0607661`).

An elliptic integrable system, ILW equation and MacDonald's difference operators

Yohei Tutiya, O-Hara Graduate School of Business, Tokyo

Abstract

The talk starts from a certain extension of the Macdonald difference operator and its Heisenberg representation. Taking the classical limit, an integro-differential evolution equation is obtained. The relation between KP hierarchy and the equation will be explained by using Sato theory.

Liouville structures

Paweł Urbański, University of Warsaw

Abstract

A *Liouville structure* is a struture isomorphic to a cotangent vector fibration. A Liouville structure is an essential ingredient of every variational formulation of a physical theory. For reasons of interpretation the Liouville structure can not be replaced by the corresponding cotangent fibration. We give a precise definition of Liouville structures, study their properties, and give examples used in variational formulations of mechanics.

Linear Stäckel matrices

Sergey Zykov, SISSA

Abstract

I would like to present the talk based on the common paper with M.V. Pavlov, G. El and other. It was shown that reductions of the kinetic equation for a dence soliton gas have wide intersection with the class of linear degenerated hydrodynamic systems of Egorov's type. The series of remarkable examples will be presented, including the new kind of solutions of the WDVV equation, which was derived from the kinetic equation.