Locally conformally symplectic bundles

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Locally conformally symplectic manifolds

Definition

A manifold M is locally conformally symplectic (LCS) if it is endowed with a non-degenerate 2-form ω and a covering $(U_\alpha)_\alpha$ such that there exist smooth functions f_α on U_α with $d(e^{-f_\alpha}\omega) = 0$.

Definition

A manifold M is LCS if there exists a non-degenerate 2-form ω and a closed 1-form θ such that $d\omega = \theta \wedge \omega$ ($d_\theta \omega = 0$, $d_\theta = d - \theta\wedge$). The closedness of θ implies $d_\theta^2 = 0$ and in fact the operator d_θ produces a cohomology, called Morse-Novikov or Lichnerowicz.

If (M, ω, θ) is an LCS manifold, then so is $(M, e^{-f}\omega, \theta + df)$. In particular, if $\theta = df$, then $e^{-f}\omega$ is symplectic and the LCS form is *globally conformally symplectic*.

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Definition

M is LCS if there exists a symplectic covering $(\tilde{M}, \Omega)$ of M with the deck group acting by homotheties of Ω.

Examples

1. The classical example is the Hopf manifold $S^1 \times S^{2n-1}$, since its universal covering is $\mathbb{C}^n \setminus \{0\}$ and its deck group $\mathbb{Z}$ acts by homotheties with respect to the symplectic form of $\mathbb{C}^n \setminus \{0\}$.
2. If (M, α) is a contact manifold, then $S^1 \times M$ is LCS with the Lee form $\theta = d\mathrm{vol}$ and the LCS form $\omega = d_\theta \alpha$.
3. The cotangent space of a manifold $\pi : T^*M \to M$ has an LCS structure given by $\theta = \pi^*\eta$, where η is any closed 1-form on M and $\omega = d_\theta \alpha$, where α is the tautological 1-form on the cotangent bundle.

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LCS bundles

Definition

A fiber bundle $F \to M \to B$ with LCS fiber (F, ω, θ) is locally conformally symplectic if its structural group reduces to diffeomorphisms preserving ω (such diffeomorphisms preserve automatically θ).

Examples

1. $S^1 \times S^3 \to S^1 \times S^7 \to S^4$
2. If G is a Lie group acting on F by preserving ω and P a G-principal bundle, then $P \times_G F$ is an LCS bundle.

Let $F_b := \pi^{-1}(b)$. Then $(F_b, \omega_b, \theta_b)$ is LCS.
We are intrested to see when the total space of such a fibration carries an LCS structure (F, Ω, Θ), which has the property that $\Omega_{|F_b} = \omega_b$ and $\Theta_{|F_b} = \theta_b$.

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In the case of a symplectic fibrations, when M is compact and the base is symplectic, we have Thurston's criterion:

Theorem

There exists a symplectic form Ω on M such that $\Omega_{|F_b} = \omega_b$ if and only if there exists $[a] \in H^2(M)$ such that $[a]_{|F_b} = [\omega_b]$.

For the LCS setting, an analogue is:

Theorem

Let $F \to M \to B$ be an LCS bundle with F compact. If there exists on M a closed 1-form Θ such that $\Theta_{|F_b} = \theta_b$, then there exists a d_Θ-closed form Ω on M with $\Omega_{|F_b} = \omega_b$ if and only if there exists a class c in the twisted cohomology $H^2_\Theta(M)$ such that for any b, $c_{|F_b} = [\omega_b]$ in $H^2_{\theta_b}(F_b)$.

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The proof rests on this result:

Let F be a compact manifold and θ a closed 1-form. If $\{\omega_t\}_t$ is a smooth family of d_θ-exact forms indexed by an open set in $\mathbb{R}^n$, then there exists a smooth family of forms $\{\psi_t\}_{t\in\mathbb{R}^n}$ such that $\omega_t = d_\theta\psi_t$ for any t.

However, as this does not solve the non-degeneracy problem, we try to extend the following result of Sternberg and Weinstein concerning symplectic fibrations.

Theorem

Let (F,ω) be a symplectic manifold with a Hamiltonian action of a Lie group G on F. If $\mu : F \to g^*$ is the momentum map, then any connection on a G-principal bundle P which is fat at all the points in $\mu(F)$ induces a symplectic form on $P \times_G F$.

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A connection form a on a principal bundle P is called *fat* in the point $f \in g^*$ if its curvature form A satisfies $f \circ A : \mathrm{Ker}a \times \mathrm{Ker}a \to \mathbb{R}$ is non-degenerate.

The idea of constructing such a form on $P \times_G F$ was to combine the symplectic structure of the fiber and the chosen connection on the principal bundle. The construction is called *coupling form*. Sternberg introduced this notion first in 1977 and he observed that by choosing any connection on P, one can construct a closed 2-form on the total space of a symplectic bundle, which restricted to the fiber is exactly its symplectic form. Later, Weinstein added the extra requirement on the connection to obtain also non-degeneracy for the coupling form.

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$\pi : P \times F \to P \times_G F$, $[p, f] := \pi(p, f)$ Let a be a connection 1-form on P and H the associated Ehresmann connection. Then H induces on $P \times_G F$ a natural connection $\mathcal{H}$. With respect to this connection, the expression of the coupling form is:

$$\Omega_{[p,f]}(X, Y) = \begin{cases} \omega_b(X, Y) & \text{if } X \text{ and } Y \text{ are vertical,} \\ 0 & \text{if } X \text{ is horizontal and } Y \text{ is vertical,} \\ \mu(f)(A_p(X', Y')) & \text{if } X \text{ and } Y \text{ horizontal.} \end{cases}$$

where X' and Y' are vectors in T_pP such that $\pi_* X' = X$ and $\pi_* Y' = Y$.

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LCS bundles

In order to extend the result of Sternberg and Weinstein to LCS manifolds, we have to adapt the notion of Hamiltonian action to the LCS case.

Definition

Let $\rho : G \to \mathrm{Diff}(M)$ be an action of G on an LCS manifold (F, ω, θ). We call it *twisted Hamiltonian* if ω is G-invariant and if $i_{\bar{v}}\omega$ is d_θ - exact, where $\bar{v}$ is the fundamental vector field of the action associated to an element v in $\mathfrak{g}$.

A choice of a smooth function $\psi : \mathfrak{g} \to \mathcal{C}^\infty(F)$ such that $i_{\bar{v}}\omega = d_\theta\psi(v)$ will give a momentum map $\mu : F \to g^*$.

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If (F, ω, θ) is not GCS, then if the action of a group G is twisted Hamiltonian, there exists a unique momentum map (which is also equivariant). The reason for that is the fact that $H^0_\theta(F, \mathbb{R})$ is precisely the obstruction for every Hamiltonian vector field to have a unique Hamiltonian associated function, as the following is an exact sequence:

$$0 \longrightarrow H^0_\theta(F, \mathbb{R}) \longrightarrow \mathcal{C}^\infty(F, \mathbb{R}) \longrightarrow \mathrm{Ham}(F) \longrightarrow 0.$$

But for θ not exact, $H^0_\theta(F, \mathbb{R})$ vanishes.

The analogue in the LCS setting of the coupling form is the following:

Theorem

Let (F, ω, θ) be an LCS manifold which is not globally conformally symplectic and let G be a Lie group acting on F twisted Hamiltonian. If $\mu : F \to g^*$ is a momentum map of the action, then any connection on a G - principal bundle P which is fat at $\mathrm{Im}\mu$ induces an LCS structure on $P \times_G F$.

The proof rests on defining the same form Ω as in the original setting.

$$\Omega_{[p,f]}(X,Y) = \begin{cases} \omega_b(X,Y) & \text{if } X \text{ and } Y \text{ are vertical,} \\ 0 & \text{if } X \text{ is horizontal and } Y \text{ is vertical,} \\ \mu(f)(A_p(X',Y')) & \text{if } X \text{ and } Y \text{ horizontal.} \end{cases}$$

However, we have to prove that there is a closed 1-form Θ on $P \times_G F$ that $d\Omega = \Theta \wedge \Omega$. Implicitely, when Θ is restricted to the fiber, we get the Lee form θ of the fiber. If $\pi : P \times F \to P \times_G F$ is the natural projection, then Θ is given by $\pi^*\Theta = (0, \theta)$, by showing first that $(0, \theta)$ is a closed 1-form which descends to $P \times_G F$.

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Twisted Hamiltonian actions

The extension of the coupling form to the LCS case shows the need to find twisted Hamiltonian actions.

Lemma

Let (M, ω, θ) be an LCS manifold with $H^1_\theta(M, \mathbb{R}) = 0$. Then the action of a group G is twisted Hamiltonian if and only if $\theta(\bar{v}) = 0$, for any fundamental vector field.

Proof.

If $\theta(\bar{v}) = 0$, $d_\theta i_{\bar{v}}\omega = di_{\bar{v}}\omega - \theta \wedge i_{\bar{v}}\omega = \mathcal{L}_{\bar{v}}\omega - i_{\bar{v}}d\omega - \theta \wedge i_{\bar{v}}\omega = -i_{\bar{v}}\theta \wedge \omega - \theta \wedge i_{\bar{v}}\omega = 0$. Conversely, if the action is twisted Hamiltonian, then $d_\theta i_{\bar{v}}\omega = 0$, hence $\mathcal{L}_{\bar{v}}\omega = \theta(\bar{v})\omega$. But $\mathcal{L}_{\bar{v}}\omega = 0$, hence $\theta(\bar{v})$. $\square$

Lemma

Let (F, ω, θ) be a connected LCS manifold with d_θ-exact LCS form, $\omega = d_\theta \eta$, and let G act on F by preserving η and θ. If G has at least one fixed point, then the action of G is twisted Hamiltonian.

Lemma

If G is a Lie group with perfect Lie algebra $\mathfrak{g}$ acting on the LCS manifold (M, ω, θ) by preserving ω and $\theta(\bar{v}) = 0$, for any v in $\mathfrak{g}$, then G acts twisted Hamiltonian.

Example

We construct twisted Hamiltonian actions on the diagonal Hopf manifolds. Let

$$S^{2n-1} = \{(z_1, z_2, \dots, z_n) \mid |z_1|^2 + \cdots + |z_n|^2 = 1\}.$$

For any collection of real numbers $a_1, \dots, a_n$, satisfying $0 < a_1 \leq a_2 \dots \leq a_n$, one can associate a contact form

$$\eta_{a_1,\dots,a_n} = \frac{1}{\sum_i a_i |z_i|^2} \eta_0$$

which is a deformation of the standard one of S^{2n-1}, $\eta_0 = \sum_i y_i dx_i - x_i dy_i$.

These contact forms will further provide a family of LCS forms on $S^1 \times S^{2n-1}$ defined by

$$\omega_{a_1,\ldots,a_n} = d_{d\mathrm{vol}}\eta_{a_1,\ldots,a_n}$$

where $d\mathrm{vol}$ is a volume form on S^1.
Let T^k be the k dimensional torus, with $k \in \{1, 2, \ldots n\}$. One can define the following action of T^k on $S^1 \times S^{2n-1}$:

$$(u_1, u_2, \ldots, u_k)\cdot(s,(z_1, z_2, \ldots, z_n)) = (s,(u_1 z_1, \ldots, u_k z_k, z_{k+1} \ldots, z_n)).$$

This action preserves $d\mathrm{vol}$ and also the contact form $\eta_{a_1,\ldots,a_n}$ for any n-tuple $a_1, \ldots, a_n$, hence the LCS form is T^k - invariant.

Moreover, $S^1 \times S^{2n-1}$ is a Vaisman manifold, therefore its twisted cohomology groups vanish. Also, $d\mathrm{vol}(\bar{v}) = 0$ is verified, so the action of T^k is twisted Hamiltonian on $S^1 \times S^{2n-1}$.

Example

Let G be a *compact* Lie group acting on M with at least one fixed point. We lift the action of G to T^*M. This action preserves the tautological 1-form of T^*M. As G is compact, we can average any closed 1-form on M and obtain a G-invariant closed one-form η. The action of G on T^*M will also preserve $\pi^*\eta$ and since G acts with at least one fixed point, any G-invariant one-form β evaluated in a fixed point p will provide a fixed point β_p for the action of G on $T^*M \Rightarrow$ twisted Hamiltonian action.

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Another approach towards finding twisted Hamiltonian actions is moving from the LCS manifold to one of its symplectic coverings. If G is a Lie group acting on M and $\tilde{M}$ is one of its coverings, one can always define on $\tilde{M}$ an action of the connected component of the identity of the universal covering of G, $\tilde{G}_0$. Intuitively, a twisted Hamiltonian action lifts to a Hamiltonian one and the reversed scenario also should work.

Theorem

Let (M, ω, θ) be an LCS manifold and $\pi : \tilde{M} \to M$ one of its symplectic coverings. If G is a Lie group acting twisted Hamiltonian on M, then $\tilde{G}_0$ acts Hamiltonian on $\tilde{M}$.

The converse is not always true. Example constructed on Inoue surface! It works, though, on some particular symplectic coverings.

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The minimal symplectic covering

Let $(\bar{M}, \Omega) \to M$ be the universal covering of M. We consider

$$\chi : \mathrm{Deck}(\bar{\mathrm{M}}/\mathrm{M}) \to \mathbb{R}$$

the function associating to an element in $\mathrm{Deck}(\bar{\mathrm{M}}/\mathrm{M})$ the corresponding conformal factor with respect to Ω.
The *minimal symplectic covering* of M is $\bar{M}/\mathrm{Ker}\chi$.

Theorem

Let $\pi : M_0 \to M$ be the minimal covering of the LCS manifold (M, ω, θ) and let Ω be the symplectic form of M_0. Let G be a Lie group acting on M by preserving ω. Then G acts twisted Hamiltonian on M if and only $\tilde{G}_0$ acts Hamiltonian on M_0.

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Thank you for your attention.