

Seria zadań z Analizy IIIC, Całki zespolone

Obliczyć całki:

1. $\int_{\partial D} \frac{dz}{1+z^4}$, $D := \{z \in \mathbb{C} \mid |z-1| < 1\}$;
2. $\int_{\partial D} \frac{z^2 \sin^2 \frac{1}{z}}{(z-1)(z-2)} dz$, $D := \{z \in \mathbb{C} \mid |z| < 3\}$;
3. $\int_{\partial D} \frac{dz}{(2+\sqrt{z-1})\sin z}$, $D := \{z \in \mathbb{C} \mid |z| < \frac{1}{2}\}$, $\sqrt{z-1}|_{z=0} = i$;
4. $\int_{\partial D} \frac{dz}{1+\ln(z-2)}$, $D := \{z \in \mathbb{C} \mid |z-3| < 0,99\}$, $\ln(z-2)|_{z=3} = 0$;
5. $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+9)}$;
6. $\int_{-\infty}^{\infty} \frac{(x+1)\sin 2x}{x^2+2x+2} dx$;
7. $\int_0^\pi \frac{\cos^4 \varphi}{1+\sin^2 \varphi} d\varphi$;
8. *v.p.* $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x-\xi)}$, $a > 0$, $\xi \in \mathbb{R}$;
9. *v.p.* $\int_0^\infty \frac{dx}{(x-1)\sqrt{x}}$;
10. $\int_0^\infty \frac{\ln x dx}{x^2-1}$;
11. $\int_0^\infty \frac{\ln^2 x dx}{(x^2+1)^2}$;
12. $\int_{-1}^1 \frac{\sqrt[5]{(1+x)(1-x)^4}}{x^2+1} dx$;
13. $\int_1^2 \frac{dx}{x\sqrt[3]{(x-1)^2(2-x)}}$;
14. *v.p.* $\int_0^\infty \frac{dx}{x^\alpha(x+a)}$, $0 < \alpha < 1$, $a > 0$;
15. $\int_0^\infty \frac{x^\alpha dx}{(x+a)(x+2a)}$, $-1 < \alpha < 1$, $a > 0$.