## CORRECTIONS AND REMARKS TO "WIGNER-TYPE THEOREMS FOR HILBERT GRASSMANNIAN"

**1. Chapter 1, Section 1.3, Remark 1.16, Page 15**. If $H$ is infinite-dimensional, then there are compatible subsets of $\mathcal{L}(H)$ which are not contained in orthogonal apartments.

The following example is proposed by Vladimir M. Kadets. Suppose that $H = L_2[0,1]$ and $\mathcal{X}$ is the set formed by all closed subspaces of type $L_2(D)$ with a Borel subset $D \subset [0,1]$. Then $\mathcal{X}$ is compatible. Let $B = \{e_k\}_{k\in\mathbb{N}}$ be an orthonormal basis of $H$. Denote by $A_k$ the support of $e_k$. There is a Borel set $D \subset [0,1]$ of positive measure such that for each $k$ the set $[0,1] \setminus D$ intersects $A_k$ in a set of positive measure, i.e. the intersection of $B$ with $L_2(D)$ is empty. Therefore, there is no orthogonal apartment containing $\mathcal{X}$.

**2. Chapter 2, Section 2.6, Page 47**. For the case when $\alpha = 2$ the characterization of the adjacency relation for complementary subsets fails. Suppose that $i, i', j, j'$ are mutually distinct indices. If $\mathcal{G} = \mathcal{G}_2(V)$, then

$$\mathcal{A}(+i,-j) \cap \mathcal{A}(+i',-j') = \mathcal{A}(+i,-j) \cap \mathcal{A}(+i',-j)$$

consists of the subspace spanned by $e_i$ and $e_{i'}$. Similarly, if $\mathcal{G} = \mathcal{G}^2(V)$, then

$$\mathcal{A}(+i,-j) \cap \mathcal{A}(+i',-j') = \mathcal{A}(+i,-j) \cap \mathcal{A}(+i,-j')$$

consists of the unique element $X \in \mathcal{A}$ such that $e_j, e_{j'} \notin X$.

So, our arguments do not work. In this case, two distinct elements of $\mathcal{A}$ are adjacent if and only if there is a complementary subset containing them. This guarantees that every spacial bijection is adjacency preserving in both directions.

**3. Chapter 4, Section 4.2, Example 4.13, Page 74**. I add some explanation for the following statement: for $Y \in \mathcal{G}_k(H) \setminus \{X\}$ we have $X' \perp A(Y)$ if and only if $X \perp Y$.

If $X \perp Y$, then $A(X) \perp A(Y)$. Since $A^*(X') = X$, the subspace $X'$ is contained in the orthogonal sum of $A(X)$ and $\mathrm{Ker}(A^*)$. The subspace $A(Y)$ is orthogonal to this sum and we obtain that $X' \perp A(Y)$.

Observe that $A(X) \subset X' + \mathrm{Ker}(A^*)$. If $X' \perp A(Y)$, then $A(X) \perp A(Y)$ which implies that $X \perp Y$.